
AP[®] AP Physics C: Electricity & Magnetism

Sample Student Responses and Scoring Commentary

Question #4

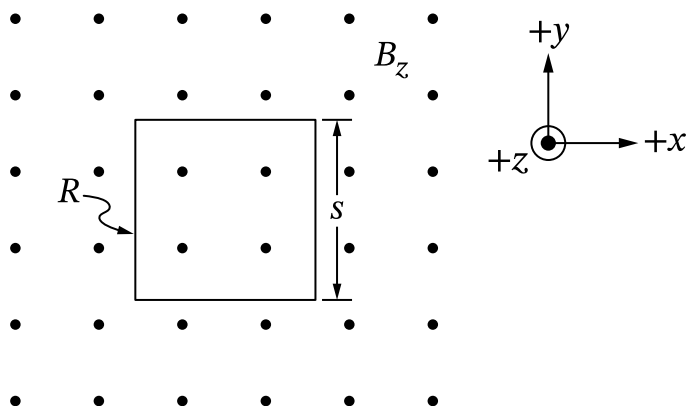
DRAFT

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Question 4

4. In Scenario 1, a square conducting loop with side length s has total resistance R . The loop is held fixed in the xy -plane in an external, uniform, time-varying magnetic field that is initially directed in the $+z$ -direction, as shown. The z -component of the magnetic field changes as a function of time t as described by $B_z = B_0 \cos\left(2\pi\frac{t}{t_1}\right)$, where B_0 and t_1 are positive constants.

**Part A**

Indicate whether the induced current I in the loop is clockwise, counterclockwise, or zero during the time interval $0 < t < \frac{t_1}{4}$ by writing one of the following.

- I is clockwise.
- I is counterclockwise.
- I is zero.

Justify your answer without only manipulating equations.

Part B

Derive an expression for I as a function of t in terms of s , R , B_0 , t_1 , t , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 2, a new square conducting loop with side length $2s$ and total resistance $2R$ is placed in the same external, uniform, time-varying magnetic field as in Scenario 1.

Indicate whether the maximum induced current I_2 in Scenario 2 is greater than, less than, or equal to the maximum induced current I_1 in Scenario 1 by writing one of the following.

- $I_2 > I_1$
- $I_2 < I_1$
- $I_2 = I_1$

Briefly **justify** your answer by referencing the expression you derived in part B.

STOP
END OF EXAM

Question 4 Qualitative Quantitative Translation (QQT)**8 points****General Scoring Notes****NR (No Response):** A score of NR is only assigned to responses that are blank.

A	For indicating that I is counterclockwise	Point A1
	For indicating that the magnitude of the magnetic field is decreasing	Point A2
	For indicating that the induced magnetic field will oppose the changing flux through the loop	Point A3

Example Response

During the time interval, the magnetic field is directed in the +z-direction and is decreasing in magnitude. To oppose this change in flux through the loop, the induced magnetic field must also be directed in the +z-direction. Therefore, a counterclockwise current is induced in the loop.

B	For a multistep derivation that includes $\Phi_B = \int \vec{B} \cdot d\vec{A}$, $\mathcal{E} = -\frac{d\Phi_B}{dt}$, or $I = \frac{\Delta V}{R}$	Point B1
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Scoring Note: Vector notation is not required for this point to be earned.

	For substituting $B_0 \cos\left(2\pi \frac{t}{t_1}\right)$ for B into an equation for magnetic flux	Point B2
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	For a correct expression (e.g., $I = \frac{2\pi s^2 B_0}{t_1 R} \sin\left(2\pi \frac{t}{t_1}\right)$)	Point B3
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Scoring Note: Sign will not be considered when awarding this point.

Example Response

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\Phi_B = B_0 \cos\left(2\pi \frac{t}{t_1}\right) s^2$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -B_0 s^2 \frac{d}{dt} \left(\cos\left(2\pi \frac{t}{t_1}\right) \right)$$

$$\mathcal{E} = \frac{2\pi s^2 B_0}{t_1} \sin\left(2\pi \frac{t}{t_1}\right)$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

$$I = \frac{2\pi s^2 B_0}{t_1 R} \sin\left(2\pi \frac{t}{t_1}\right)$$

C	For indicating $I_2 > I_1$	Point C1
	For a justification that correctly references the functional dependence of the current on s and R in the expression derived in part B	Point C2
	Example Response $I_2 > I_1$ <i>The side length and the resistance of the loop doubles. The derivation in part B shows that the current is proportional to the square of the side length and inversely proportional to R, so the current increases.</i>	

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 4

PART A

$\vec{I}$ is counterclockwise.

- According to B_z , the B field is decreasing in the $+z$ direction in the interval $0 < t < \frac{t_1}{4}$, so the net magnetic flux is in the $-z$ direction.
- According to Lenz's law, the induced current creates a B field opposite of the change in $\vec{B}$, so the induced current creates B field in the $+z$ direction.
- By Right hand rule, the induced current is counterclockwise.

PART B

$$\Phi_B = B A \cos \theta$$

$$= B_0 \cos\left(2\pi \frac{t}{t_1}\right) \cdot s^2$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

$$= -B_0 s^2 \sin\left(2\pi \frac{t}{t_1}\right) \cdot \frac{2\pi}{t_1}$$

$$= -\frac{B_0 s^2 2\pi}{t_1} \sin\left(2\pi \frac{t}{t_1}\right)$$

$$I = \frac{\mathcal{E}}{R}$$

$$= \frac{2\pi B_0 s^2 \sin\left(2\pi \frac{t}{t_1}\right)}{t_1 R}$$

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Question 4

PART C

$I_2 > I_1$, according to derivation in part b, $I \propto s^2$
and $I \propto \frac{1}{R}$,

- Doubling s increases I by a factor of 4
- Doubling R decreases I by a factor of 2
- So the maximum I generated is double the amount as case 1

$$I_{\max} = \frac{2\pi B_0 s^2}{t_1 R}$$

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Question 4

PART A

$\vec{I}$ is counterclockwise

because B_z is decreasing from $0 < t < \frac{t_1}{4}$

which means B_{ind} is out of the plane

which, by the right-hand rule, means I is counterclockwise

PART B

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A \frac{dB}{dt}$$

$$\mathcal{E} = -s^2 \frac{d}{dt} (B_0 \cos(2\pi \frac{t}{t_1}))$$

$$\mathcal{E} = -\frac{2\pi s^2 B_0}{t_1} \sin(2\pi \frac{t}{t_1})$$

$$\mathcal{E} = IR \Rightarrow I = \frac{\mathcal{E}}{R}$$

$$I = \frac{2\pi s^2 B_0}{t_1 R} \sin(2\pi \frac{t}{t_1})$$

where positive
is counterclockwise current.

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Question 4

PART C

$$I_2 > I_1$$

By equation B,
$$I_{\max} = \frac{2\pi s^2 B_0}{t, R}$$

meaning that doubling s quadruples $I_{\max}$
and doubling R halves $I_{\max}$

$$4 \times \frac{1}{2} = 2$$

so, overall, $I_{\max}$ will increase
in scenario 2.

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Question 4

PART A

I is counterclockwise.

We know that the $\vec{B}$ field is varying in the $+z$ direction. Using the Right hand rule we can see the direction of I is counterclockwise. We also know that $\Delta\Phi_B \propto \mathcal{E}$ and ohm's law tells us $V=IR \Rightarrow V \propto I$ so as B changes Φ_B changes and hence a emf is induced.

PART B

$$\frac{d\Phi_B}{dt} = -\mathcal{E} \quad \mathcal{E} = IR \quad A = S^2$$

$$\Rightarrow \frac{d}{dt} \{ \vec{B} \cdot \vec{A} \} = -IR$$

$$\Rightarrow I = -\frac{S^2}{R} \frac{d}{dt} \{ B_0 \cos(2\pi \frac{t}{t_1}) \}$$

$$\Rightarrow I = -\frac{S^2}{R} \left(B_0 \left(\frac{2\pi}{t_1} \right) (-\sin(2\pi \frac{t}{t_1})) \right)$$

$$\Rightarrow \boxed{I = \frac{2S^2 B_0 \pi}{R t_1} \sin(2\pi \frac{t}{t_1})}$$

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Question 4

PART C

$$\underline{I_2 > I_1}$$

The current of the loop, according to part B's derivation, is $\propto s^2$ and $\propto \frac{1}{R}$. Since the value of s is squared the ratio of the $\vec{I}$ is going to be $\frac{(2s)^2}{2R} : \frac{(s)^2}{R} :: I_2 : I_1$. Since the new loop's value for $\frac{s^2}{R}$ is bigger $I_2 > I_1$.

$$I_2 = \frac{4s^2 B_0 \pi}{R t_1} \sin\left(2\pi \frac{t}{t_1}\right)$$

$$I_1 = \frac{2s^2 B_0 \pi}{R t_1} \sin\left(2\pi \frac{t}{t_1}\right)$$

(Also $I_2 > I_1$ because the $\vec{B}$ is the same for I_1 and I_2)

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Question 4

PART A

I is going clockwise. If the $\vec{B}$ field out of the page makes a count-clockwise loop by right hand rule, then the induced current by Lenz law is opposing that which is going clockwise.

PART B

$$V = IR$$

$$\frac{d\Phi}{dt} = IR$$

$$\Phi = BA \cos \theta$$

$$\Phi = B \cdot s^2 = B_0 s^2 \cos\left(\frac{2\pi t}{t_1}\right)$$

$$\begin{aligned} \frac{d\Phi}{dt} &= \frac{d}{dt} B_0 s^2 \cos\left(\frac{2\pi t}{t_1}\right) \\ &= -B_0 s^2 \cdot \frac{2\pi}{t_1} \cdot \sin\left(\frac{2\pi t}{t_1}\right) \end{aligned}$$

$$IR = -\frac{2\pi B_0 s^2}{t_1} \cdot \sin\left(\frac{2\pi t}{t_1}\right)$$

$$I = \frac{-2\pi B_0 s^2 \sin\left(\frac{2\pi t}{t_1}\right)}{t_1 R}$$

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Question 4**PART C**

$$I_2 > I_1$$

The maximum induced current would increase since I depends on s from part (B) and would therefore see an increase in the maximum current as a result since the value of s is in the numerator so if s increases, so does I .

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Question 4

PART A

$$B_z = B_0 \cdot \cos\left(2\pi \cdot \frac{t}{T_1}\right)$$

The induced current I in the loop must be counter clockwise during the time interval $0 < t \leq \frac{T_1}{4}$ because of the right hand rule and due to the magnetic field being positive during this time period.

PART B

$$\oint \mathbf{B}_z \cdot d\mathbf{l} = \mu_0 \cdot I_{enc}$$

$$B_z = B_0 \cdot \cos\left(2\pi \cdot \frac{t}{T_1}\right) \text{ when } I_{enc}$$

$$B_0 \cdot \left(\oint \cos\left(2\pi \cdot \frac{t}{T_1}\right) d\mathbf{l} \right) = \mu_0 \cdot I_{enc}$$

$$\frac{B_0 \cdot s^2}{R} \int_0^t \cos\left(2\pi \cdot \frac{t}{T_1}\right) \cdot dt = \mu_0 \cdot I_{enc}$$

$$B_0 \cdot s^2 \cdot \frac{\sin\left(2\pi \left(\frac{t}{T_1}\right)\right)}{2\pi R} = \mu_0 \cdot I_{enc}$$

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Question 4

PART C

$$\frac{B_0 B_0 \cdot s^2 \cdot \frac{\sin(2\pi \frac{t}{T_1})}{2\pi R}}{2\pi R} = \mu_0 \cdot I_{enc}$$

If we increased the area of the conducting loop and increased the total resistance to $2s$ and $2R$ then the maximum induced current in scenario 2 must be greater than in scenario 1.

This is because the area of the conducting loop increases by a much larger size compared to the resistance so it must have a greater current.

$$\frac{B_0 \cdot s^2 \cdot \sin(2\pi \frac{t}{T_1})}{2\pi R} < \frac{4s^2 \cdot B_0 \cdot \sin(\pi \frac{t}{T_1})}{4\pi R}$$

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Question 4

PART A

I is counter clockwise due to the right hand rule, and the magnetic field being out of the page

PART B

$$I(t) = \frac{B_0 l}{\mu}$$

$$l = 5 \quad B_z = B_0 \cos(2\pi \frac{t}{T_0})$$

$$I(t) = \frac{B_0 s}{\mu}$$

$$\mu = \frac{B}{nI}$$

$$I(t) = \frac{B_0 s n}{B}$$

$$I(t) = \frac{s n B}{t R}$$

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Question 4**PART C**

$$I_2 = I_1$$

The increased side length by $2\times$ will
offset with increased Resistance
by $2\times$, therefore $I_2 = I_1$

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Question 4

Sample Identifier: PCEM Sample 4A

Score: 8

- Point A1 was earned. The response indicates the induced current is counterclockwise.
- Point A2 was earned. The response indicates " B field is decreasing".
- Point A3 was earned. The response indicates "the induced current creates a B field opposite of the change in Φ_B ."
- Point B1 was earned. The response uses Faraday's law, Ohm's law, or magnetic flux in a multistep derivation.
- Point B2 was earned. The response substitutes the magnetic field function into an equation for magnetic flux.
- Point B3 was earned. The response indicates a correct expression for the induced current.
- Point C1 was earned. The response indicates that $I_2 > I_1$.
- Point C2 was earned. The response indicates the correct functional dependence of the current on s and R in the expression derived in part B.

Sample Identifier: PCEM Sample 4B

Score: 7

- Point A1 was earned. The response indicates I is counterclockwise.
- Point A2 was earned. The response indicates " B field is decreasing".
- Point A3 was not earned. The response does not include any reference to the induced magnetic field opposing the changing flux.
- Point B1 was earned. The response uses Faraday's law, Ohm's law, or magnetic flux in a multistep derivation.
- Point B2 was earned. The response substitutes the magnetic field function into an equation for magnetic flux within Faraday's law.
- Point B3 was earned. The response indicates a correct expression for the induced current.
- Point C1 was earned. The response indicates that $I_2 > I_1$.
- Point C2 was earned. The response indicates the correct functional dependence of the current on s and R in the expression derived in part B.

Sample Identifier: PCEM Sample 4C**Score: 6**

- Point A1 was earned. The response indicates the induced current is counterclockwise.
- Point A2 was not earned. The response does not indicate the magnetic field is decreasing.
- Point A3 was not earned. The response does not include any reference to the induced magnetic field opposing the changing flux.
- Point B1 was earned. The response uses Faraday's law, Ohm's law, or magnetic flux in a multistep derivation.
- Point B2 was earned. The response substitutes the magnetic field function into an equation for magnetic flux within Faraday's law.
- Point B3 was earned. The response indicates a correct expression for induced current.
- Point C1 was earned. The response indicates that $I_2 > I_1$.
- Point C2 was earned. The response indicates the correct functional dependence of the current on s and R in the expression derived in part B.

Sample Identifier: PCEM Sample 4D**Score: 4**

- Point A1 was not earned. The response indicates the induced current is clockwise, instead of counterclockwise.
- Point A2 was not earned. The response does not indicate the magnetic field is decreasing.
- Point A3 was not earned. The response does not connect "opposing" to induced magnetic field and changing flux.
- Point B1 was earned. The response uses Faraday's law, Ohm's law, or magnetic flux in a multistep derivation.
- Point B2 was earned. The response substitutes the magnetic field function into an equation for magnetic flux.
- Point B3 was earned. The response indicates a correct expression for the induced current.
- Point C1 was earned. The response indicates that $I_2 > I_1$.
- Point C2 was not earned. The response indicates the correct functional dependence of the current on s , but does not reference the relationship between current and resistance.

Sample Identifier: PCEM Sample 4E**Score: 3**

- Point A1 was earned. The response indicates the induced current is counterclockwise.
- Point A2 was not earned. The response does not indicate the magnetic field is decreasing.
- Point A3 was not earned. The response does not include any reference to the induced magnetic field opposing the changing flux.
- Point B1 was not earned. The response does not include magnetic flux, Faraday's law, or Ohm's law in the multistep derivation.
- Point B2 was not earned. The response does not substitute the function for magnetic field into a magnetic flux equation.
- Point B3 was not earned. The response indicates an incorrect expression for induced current.
- Point C1 was earned. The response indicates "the induced current in scenario 2 must be greater than in scenario 1".
- Point C2 was earned. The response indicates the correct functional dependence of the current on s and R in the expression derived in part B.

Sample Identifier: PCEM Sample 4F**Score: 2**

- Point A1 was earned. The response indicates the induced current is counterclockwise.
- Point A2 was not earned. The response does not indicate the magnetic field is decreasing.
- Point A3 was not earned. The response does not include any reference to the induced magnetic field opposing the changing flux.
- Point B1 was not earned. The response does not include magnetic flux, Faraday's law, or Ohm's law in the multistep derivation.
- Point B2 was not earned. The response does not substitute the function for magnetic field into a magnetic flux equation.
- Point B3 was not earned. The response indicates an incorrect expression for the induced current.
- Point C1 was not earned. The response indicates $I_2 = I_1$.
- Point C2 was earned. The response indicates the correct functional dependence of the current on s and R in the expression derived in part B.