
AP[®] Physics C: Mechanics

Sample Student Responses and Scoring Commentary

Question #4

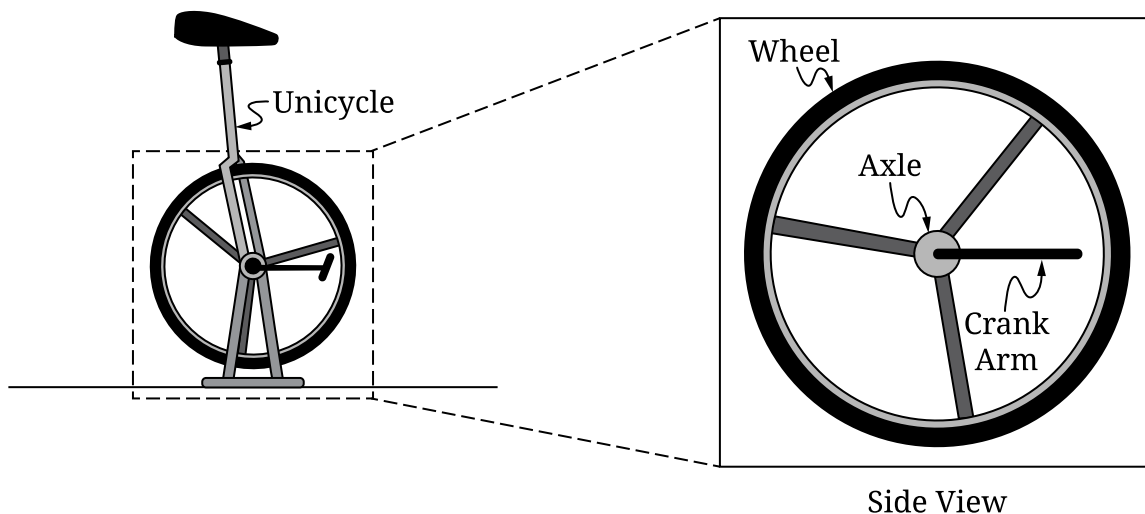
DRAFT

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Question 4

4. A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_W . One end of a crank arm can be attached to the wheel, as shown in the side view.



In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1 . The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1 .
- Scenario 2: The length of the crank arm is ℓ_2 , where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2 .

Part A

Indicate whether ω_2 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_2 > \omega_1$
- $\omega_2 < \omega_1$
- $\omega_2 = \omega_1$

Justify your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Part B

Derive an expression for the angular speed ω_1 of the wheel in Scenario 1 immediately after the crank arm has been turned through one rotation. Express your answer in terms of I_W , F , ℓ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

In Scenario 3, the unicycle is placed on the ground such that the unicycle can move forward without the wheel slipping on the ground. A force of the same magnitude F is exerted at the end of the crank arm of length ℓ_1 , as in Scenario 1. The direction of the force remains perpendicular to the crank arm at all times. The resulting angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_3 .

Indicate whether ω_3 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_3 > \omega_1$
- $\omega_3 < \omega_1$
- $\omega_3 = \omega_1$

Briefly **justify** your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

STOP
END OF EXAM

Question 4 Qualitative Quantitative Translation (QQT)**8 points****General Scoring Notes****NR (No Response):** A score of NR is only assigned to responses that are blank.

A	For indicating $\omega_2 > \omega_1$	Point A1
	For a justification that correctly relates force and crank arm length to the torque exerted on the crank arm	Point A2
	For a justification that correctly relates torque exerted over a single rotation to one of the following:	Point A3
	- the rotational kinetic energy of the wheel - the angular acceleration	
Example Response		
$\omega_2 > \omega_1$. The torque exerted on the crank arm will be greater at the end of the crank arm with a greater length, and a greater torque exerted over one rotation means a greater change in rotational kinetic energy of the wheel. A greater rotational kinetic energy for the same wheel with the same rotational inertia means the wheel has a greater angular speed.		
B	For a multistep derivation that includes one of the following:	Point B1
	- conservation of energy that indicates work is done on the wheel - the work-energy theorem - Newton's second law in rotational form	
	For indicating one of the following:	Point B2
	- the work done by F on the crank arm is $2\pi F\ell_1$ - the magnitude of the angular acceleration of the wheel is $\alpha = \frac{F\ell_1}{I_W}$	
For a correct expression for ω_1 in terms of given quantities		Point B3
Example Responses		
Solution 1 $W = \Delta K$ $\tau\Delta\theta = \Delta K_{\text{rot}}$ $(F\ell_1)(2\pi) = \frac{1}{2}I\omega_1^2$ $\frac{4\pi F\ell_1}{I_W} = \omega_1^2$ $\omega_1 = \sqrt{\frac{4\pi F\ell_1}{I_W}}$ Solution 2 $\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}}$ $\alpha = \frac{Fr}{I} = \frac{F\ell_1}{I_W}$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\omega_1^2 = 2\alpha(2\pi)$ $\omega_1^2 = 4\pi\alpha$ $\omega_1^2 = 4\pi \frac{F\ell_1}{I_W}$ $\omega_1 = \sqrt{\frac{4\pi F\ell_1}{I_W}}$		

C	For indicating $\omega_3 < \omega_1$	Point C1
	For a justification that indicates one of the following:	Point C2
	- the work done to turn the crank arm is converted into translational and rotational kinetic energies - the torque exerted on the wheel by friction changes the net torque on the wheel	
	Example Response	
	<i>$\omega_3 < \omega_1$. When the unicycle is stationary, the work done by the force of magnitude F to turn the crank arm one rotation is converted into only rotational kinetic energy. When the unicycle can move forward, the same amount of work done is converted into translational and rotational kinetic energies. This means the wheel has to turn more slowly when the unicycle can move forward.</i>	

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Question 4

PART A

$$\alpha = \frac{\sum \tau}{I}$$

ω_2 is greater than ω_1 , because the crank arm length increased, so the torque and angular acceleration increased, which means the rotational velocity after one rotation is also greater

$$\omega_2 > \omega_1$$

PART B

$$\alpha = \frac{\sum \tau}{I}$$

$$\alpha = \frac{F l_1}{I_w}$$

$$\omega^2 = \omega_o^2 + 2\alpha \Delta\theta$$

$$\omega^2 = 0 + 4\pi \frac{F l_1}{I_w}$$

$$\omega = 2\sqrt{\frac{\pi F l_1}{I_w}}$$

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Question 4**PART C**

$$\alpha = \frac{\sum \tau}{I}$$

$$\omega_3 < \omega_1$$

ω_3 is less than ω_1 because now that the wheel is on the ground, there is a torque from static friction that opposes the force applied to the crank arm. Therefore, after one full rotation, the angular velocity will decrease,

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Question 4

PART A

$$I_1 = I_2 = I_w$$

$$\omega_2 > \omega_1$$

$$\tau_2 = l_2 F \quad \tau_1 = l_1 F \Rightarrow \tau_2 > \tau_1$$

By the work-energy theorem:

$$\Delta E = W \quad E_0 = 0 \quad E_{\text{after}} = W$$

$$E_{\text{after}} = W = \tau \cdot \theta = \tau 2\pi$$

Energy is stored in rotational kinetic energy:

$$E_{\text{after}} = \frac{1}{2} I \omega^2 \quad 2\pi \tau = \frac{1}{2} I \omega^2$$

$$\tau_2 > \tau_1 \Rightarrow \omega_2^2 > \omega_1^2 \Rightarrow \omega_2 > \omega_1$$

More work is done in Scenario 2 because the torque is greater because $l_2 > l_1$. Thus, since all energy is stored in rotational KE, and θ is 2π , $\omega_2 > \omega_1$.

PART B

$$\tau_1 = |\vec{r} \times \vec{F}| = l_1 F$$

$$2\pi \tau_1 = \frac{1}{2} I_w \omega_1^2$$

$$4\pi l_1 F = I_w \omega_1^2$$

$$\omega_1 = 2 \sqrt{\frac{\pi l_1 F}{I_w}}$$

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Question 4

PART C

$$\omega_3 > \omega_1$$

Since the wheel does not slip, there is a force F_f exerted by the ground on the wheel, which produces a torque τ_f in the same direction as τ_1 , the torque of the crank arm. Therefore, the net torque is greater than scenario 1, so the total work and thus energy of the system is greater, leading to a higher final angular velocity ω_3 since all energy is stored in rotational kinetic energy and angular displacement is the same $\theta = 2\pi$.

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Question 4

PART A

$$\omega_2 > \omega_1$$

more work is done in scenario 2, because the force is applied at a greater distance. This means scenario 2 has more energy and is spinning faster.

PART B

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = 2\pi$$

$$\omega_0 = 0$$

$$\alpha = \frac{\tau}{I}$$

$$\alpha = \frac{F\ell_1}{I}$$

$$\boxed{\omega_1 = 2\sqrt{\pi \frac{F\ell_1}{I_w}}}$$

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Question 4**PART C**

$$\omega_3 < \omega_1$$

The same total work is done in both scenarios. In scenario 1, all energy becomes rotational kinetic energy. In scenario 2, some of the energy becomes translational kinetic energy. Scenario 1 will have more rotational energy so it will be spinning faster.

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Question 4

PART A

W2X0, since the crank arm is longer in scenario 2, you are applying more torque for the same $\Delta\theta$ about the rotational axis than in scenario 1, as a result, the angular speed will be greater for the second one.

PART B

$$\tau_{\text{net}} = I\alpha$$

$$\tau_{\text{net}} = I \frac{d\omega}{dt}$$

$$Fl =$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$$

$$\alpha = \frac{\tau}{I} = \frac{Fl}{I_w}$$

$$F = ma$$

$$\alpha = \frac{F}{m}$$

$$\omega_f = \sqrt{2 \frac{Fl}{I_w} 2\pi}$$

$$= \sqrt{\frac{4Fl}{I_w} \pi}$$

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Question 4

PART C

$$v = r\omega$$

$$a_T = r\alpha$$

$$\omega_3 = \omega_1$$

Since the same force is being applied for the same Δt , with the same torque, the angular ~~acceleration~~ the wheel remains the same

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Question 4

PART A

$$\omega_1 = \omega_2$$

the angular speeds are the same b/c the same amount of θ is displacement over the same times. therefore, $\frac{\theta}{t}$ remains the same, keeping the ω the same. the linear velocities are different.

PART B

$$\alpha = \frac{\tau}{I_w}$$

$$\tau = r \times F \rightarrow r \sin(\theta) \cdot F$$

$$\tau = l_1 \cdot F \quad \sim 90^\circ \text{ b/c } \perp$$

$$\alpha = \frac{l_1 \cdot F}{I_w}$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f = \sqrt{2 \left(\frac{l_1 \cdot F}{I_w} \right) (2\pi)}$$

$$\text{adp} \quad \omega_i = \sqrt{4\pi \left(\frac{l_1 \cdot F}{I_w} \right)}$$

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Question 4

PART C

$$\omega_3 > \omega_1$$

some of the rotational momentum is lost to as heat b/c of friction. as a result, it is easier to rotate the wheel. this results in a faster ω_3 . $I\omega$ decreases, resulting in $\frac{L_1 \cdot F}{I\omega}$ to be divided by a smaller number. this increases $\frac{L_1 \cdot F}{I\omega}$, which will result in a larger ω_3 .

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Question 4

PART A

$\omega_2 > \omega_1$, because the force exerted on each arm is the same but the radius isn't which is directly proportional to calculating the torque ($\tau = r \times F$), since L_2 has a greater radius, it will also have a greater torque, therefore meaning its angular speed will be greater because there was more torque exerted

PART B

$$K_{rot} = \frac{1}{2} I \omega^2$$

$$W = \int \tau d\theta$$

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Question 4

PART C

$\omega_3 = \omega_1$ because the only difference is in scenario 2, the unicycle is on the ground, however the ground is frictionless therefore there is no external non conservative forces acting on the system and due to the conservation of energy that means $K E_{rot} = \frac{1}{2} I \omega^2$ will be equivalent in both scenarios. This matters because ω /angular speed is in the equation

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Qualitative-Quantitative Translation (QQT)

Sample: 4A

QQT Part A Point 1 Score: 1

QQT Part A Point 2 Score: 1

QQT Part A Point 3 Score: 1

QQT Part B Point 1 Score: 1

QQT Part B Point 2 Score: 1

QQT Part B Point 3 Score: 1

QQT Part C Point 1 Score: 1

QQT Part C Point 2 Score: 1

Score: 8

A.

- Point A1 was earned. The response indicates that $\omega_2 > \omega_1$.
- Point A2 was earned. The response correctly relates force and crank arm length to the torque exerted on the crank arm.
- Point A3 was earned. The response correctly relates torque exerted over a single rotation to the angular acceleration.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law in rotational form.
- Point B2 was earned. The response indicates the correct magnitude of the angular acceleration of the wheel.
- Point B3 was earned. The response includes a correct expression for ω_1 in terms of given quantities.

C.

- Point C1 was earned. The response indicates that $\omega_3 < \omega_1$.
- Point C2 was earned. The response indicates that the torque exerted on the wheel by friction changes the net torque on the wheel.

Sample: 4B

QQT Part A Point 1 Score: 1

QQT Part A Point 2 Score: 1

QQT Part A Point 3 Score: 1

QQT Part B Point 1 Score: 1

QQT Part B Point 2 Score: 1

QQT Part B Point 3 Score: 1

QQT Part C Point 1 Score: 0

QQT Part C Point 2 Score: 1

Score: 7

A.

- Point A1 was earned. The response indicates that $\omega_2 > \omega_1$.
- Point A2 was earned. The response correctly relates force and crank arm length to the torque exerted on the crank arm.
- Point A3 was earned. The response correctly relates torque exerted over a single rotation to the rotational kinetic energy of the wheel.

B.

- Point B1 was earned. The response includes a multistep derivation that includes the work-energy theorem.
- Point B2 was earned. The response indicates the work done by F on the crank arm is $2\pi F \ell_1$. The response multiplies the expression by 2, eliminating the $\frac{1}{2}$ from the energy term, which also makes the work term $4\pi F \ell_1$.
- Point B3 was earned. The response includes a correct expression for ω_1 in terms of given quantities.

C.

- Point C1 was not earned. The response indicates that $\omega_3 > \omega_1$.
- Point C2 was earned. The response indicates that the torque exerted on the wheel by friction changes the net torque on the wheel.

Sample: 4C

QQT Part A Point 1 Score: 1

QQT Part A Point 2 Score: 0

QQT Part A Point 3 Score: 0

QQT Part B Point 1 Score: 1

QQT Part B Point 2 Score: 1

QQT Part B Point 3 Score: 1

QQT Part C Point 1 Score: 1

QQT Part C Point 2 Score: 1

Score: 6

A.

- Point A1 was earned. The response indicates that $\omega_2 > \omega_1$.
- Point A2 was not earned. The response does not clearly relate the crank arm length to torque. Lacking this connection, it is not clear whether the response incorporates distance as a lever arm or a component of linear work.
- Point A3 was not earned. The response does not clearly relate torque to the rotational kinetic energy of the wheel. While the response does connect work to energy, without the concept of torque this is not sufficient to earn the point.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law in rotational form.
- Point B2 was earned. The response indicates the correct magnitude of the angular acceleration of the wheel.
- Point B3 was earned. The response includes a correct expression for ω_1 in terms of given quantities.

C.

- Point C1 was earned. The response indicates that $\omega_3 < \omega_1$.
- Point C2 was earned. The response indicates that the work done to turn the crank arm is converted into translational and rotational kinetic energies.

Sample: 4D

QQT Part A Point 1 Score: 1

QQT Part A Point 2 Score: 1

QQT Part A Point 3 Score: 0

QQT Part B Point 1 Score: 1

QQT Part B Point 2 Score: 1

QQT Part B Point 3 Score: 1

QQT Part C Point 1 Score: 0

QQT Part C Point 2 Score: 0

Score: 5

A.

- Point A1 was earned. The response indicates that $\omega_2 > \omega_1$.
- Point A2 was earned. The response correctly relates force and crank arm length to the torque exerted on the crank arm.
- Point A3 was not earned. The response does not relate torque to the rotational kinetic energy of the wheel.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law in rotational form.
- Point B2 was earned. The response includes the correct magnitude of the angular acceleration of the wheel.
- Point B3 was earned. The response includes a correct expression for ω_1 in terms of given quantities.

C.

- Point C1 was not earned. The response indicates that $\omega_3 = \omega_1$.
- Point C2 was not earned. The response does not indicate that the torque exerted on the wheel by friction changes the net torque on the wheel.

Sample: 4E

QQT Part A Point 1 Score: 0

QQT Part A Point 2 Score: 0

QQT Part A Point 3 Score: 0

QQT Part B Point 1 Score: 1

QQT Part B Point 2 Score: 1

QQT Part B Point 3 Score: 1

QQT Part C Point 1 Score: 0

QQT Part C Point 2 Score: 0

Score: 3

A.

- Point A1 was not earned. The response indicates that $\omega_2 = \omega_1$.
- Point A2 was not earned. The response does not correctly relate force and crank arm length to the torque exerted on the crank arm.
- Point A3 was not earned. The response does not address either torque or work done.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law in rotational form.
- Point B2 was earned. The response includes the correct magnitude of the angular acceleration of the wheel.
- Point B3 was earned. The response includes a correct expression for ω_1 in terms of given quantities.

C.

- Point C1 was not earned. The response indicates that $\omega_3 > \omega_1$.
- Point C2 was not earned. The response does not utilize either torque or energy arguments to describe the motion of the wheel.

Sample: 4F

QQT Part A Point 1 Score: 1

QQT Part A Point 2 Score: 1

QQT Part A Point 3 Score: 0

QQT Part B Point 1 Score: 0

QQT Part B Point 2 Score: 0

QQT Part B Point 3 Score: 0

QQT Part C Point 1 Score: 0

QQT Part C Point 2 Score: 0

Score: 2

A.

- Point A1 was earned. The response indicates that $\omega_2 > \omega_1$.
- Point A2 was earned. The response correctly relates force and crank arm length to the torque exerted on the crank arm.
- Point A3 was not earned. The response does not relate torque to the angular acceleration.

B.

- Point B1 was not earned. The response includes expressions for work and kinetic energy, but does not connect those in the work-energy theorem.
- Point B2 was not earned. The response does not include the correct magnitude of the work done by F on the crank arm.
- Point B3 was not earned. The response does not include a correct expression for ω_1 in terms of given quantities.

C.

- Point C1 was not earned. The response indicates that $\omega_3 = \omega_1$.
- Point C2 was not earned. The response does not indicate that the work done to turn the crank arm is converted into translational and rotational kinetic energies.