
AP[®] Physics C: Mechanics

Sample Student Responses and Scoring Commentary

Question #2

DRAFT

These exam materials may **not** be posted on school or personal websites, nor electronically redistributed for any reason without the express permission of College Board.

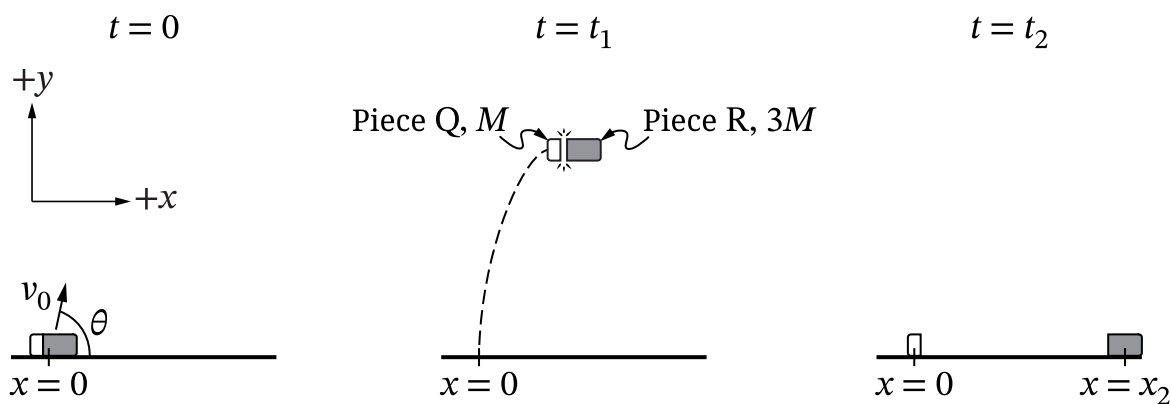
These training materials may be used solely for noncommercial purposes by AP teachers and students for course exam preparation. Permission for any other use must be sought from College Board. Teachers may reproduce these training materials in whole or in part, in limited quantities, for noncommercial, face-to-face teaching purposes, as long as the copyright notices are kept intact.

Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

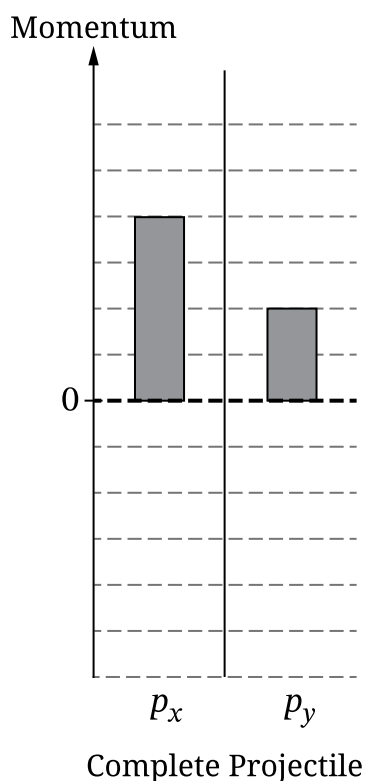
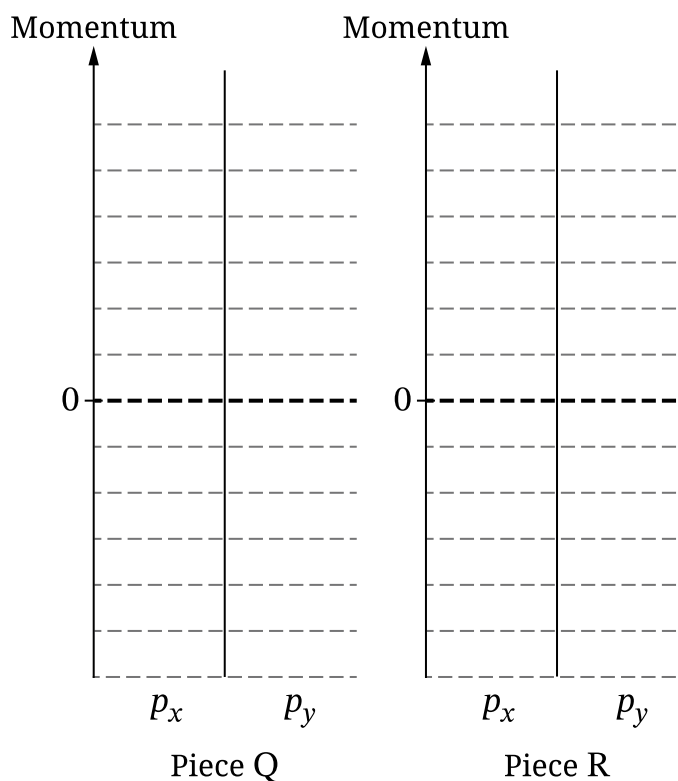
- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1

Note: Figure not drawn to scale.

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y , respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2 $t = 0$ **Figure 3** $t = t_1$ 

On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

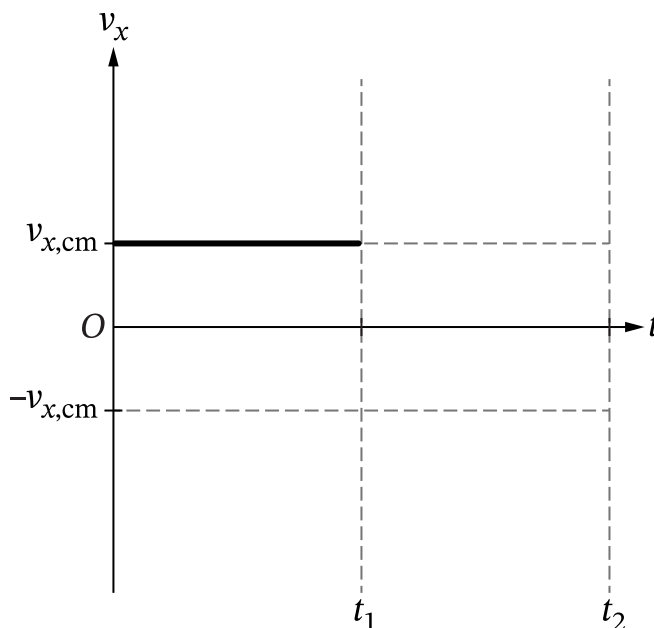
- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

Part B

Derive an expression for x_2 in terms of v_0 , θ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x . Figure 4 shows the horizontal component $v_{x,\text{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4

On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

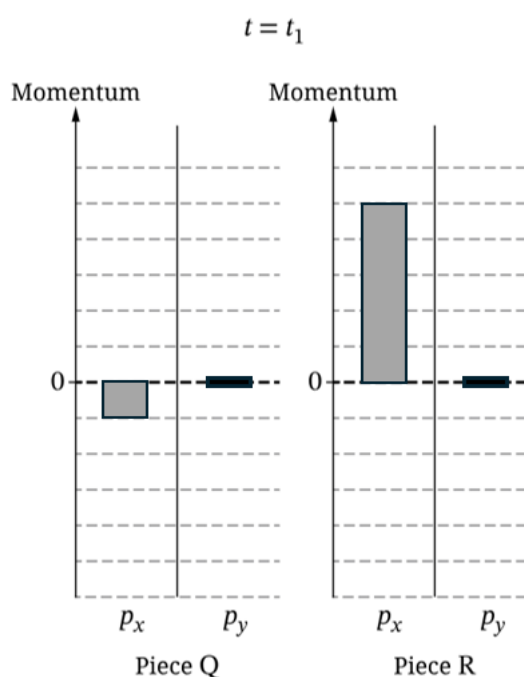
Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\text{new}} > x_2$
- $x_{\text{new}} < x_2$
- $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

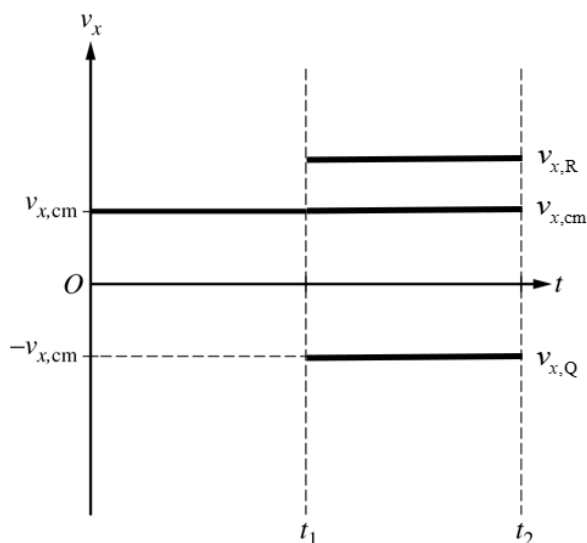
Question 2 Translation Between Representations (TBR)**12 points****General Scoring Notes****NR (No Response):** A score of NR is only assigned to responses that are blank.

A	For drawing bars for p_x of Pieces Q and R that have heights which add to 4 units	Point A1
	For drawing lines on the zero-momentum line for p_y of Pieces Q and R	Point A2
	For drawing one of the following: - A bar for p_x of Piece Q with a height of -1 unit - A bar for p_x of Piece R with a height of 5 units	Point A3

Example Response

B	For a multistep derivation that includes one of the following: - Any kinematics equation relevant to constant acceleration - Conservation of momentum	Point B1
	For indicating one of the following: - that the vertical component of the velocity of the projectile is zero at $t = t_1$ - that the vertical displacement of the projectile is zero at $t = t_2$	Point B2
	For one of the following: - correctly using conservation of momentum to determine the horizontal component of the velocity of Piece R immediately after the projectile breaks apart - correctly indicating the position of Piece R relative to the position of the center of mass of the projectile at $t = t_2$	Point B3
	For a correct expression for x_2 in terms of given quantities	Point B4
Example Responses		
<i>Solution 1</i> $\Delta y = v_{y0}t + \frac{1}{2}at^2$ $v_{y0} = v_0 \sin \theta$ $0 = v_0 \sin \theta t_2 + \frac{1}{2}(-g)t_2^2$ $t_2 = \frac{2v_0 \sin \theta}{g}$ $\sum \vec{p}_{0x} = \sum \vec{p}_{fx}$ $\vec{p}_{0x} = \vec{p}_{Qx} + \vec{p}_{Rx}$ $4Mv_0 \cos \theta = -Mv_0 \cos \theta + 3Mv_{Rf}$ $5Mv_0 \cos \theta = 3Mv_{Rf}$ $v_{Rf} = \frac{5}{3}v_0 \cos \theta$ $x_2 = x_{\text{proj}} + x_R$ $x_2 = v_{x0}t_{\text{up}} + v_{Rf}t_{\text{down}}$ $x_2 = (v_0 \cos \theta)\left(\frac{v_0 \sin \theta}{g}\right) + \left(\frac{5}{3}v_0 \cos \theta\right)\left(\frac{v_0 \sin \theta}{g}\right)$ $x_2 = \frac{v_0^2}{g} \cos \theta \sin \theta + \frac{5}{3} \frac{v_0^2}{g} \cos \theta \sin \theta$ $x_2 = \frac{8v_0^2}{3g} \cos \theta \sin \theta$ <i>Solution 2</i> $v_y = v_{y0} + a_y t$ $0 = v_0 \sin \theta + (-g)t_{\text{up}}$ $t_1 = \frac{v_0 \sin \theta}{g}$ $t_2 = 2t_1 = \frac{2v_0 \sin \theta}{g}$ $v_x = v_0 \cos \theta$ $x_{\text{CM}} = v_x t_2$ $x_{\text{CM}} = \frac{2v_0^2 \cos \theta \sin \theta}{g}$ $x_{\text{CM}} = \frac{M(0) + 3M(x_2)}{M + 3M}$ $x_{\text{CM}} = \frac{3}{4}x_2$ $\frac{3}{4}x_2 = \frac{2v_0^2 \cos \theta \sin \theta}{g}$ $x_2 = \frac{8v_0^2}{3g} \cos \theta \sin \theta$		

C	For sketching a labeled horizontal line for the center of mass that has a value equal to $v_{x,\text{cm}}$	Point C1
Scoring Note: Any clearly defined labels may earn this point.		
	For sketching a labeled horizontal line for Piece R that has a value greater than $v_{x,\text{cm}}$	Point C2
	For sketching a labeled horizontal line for Piece Q that has a value equal to $-v_{x,\text{cm}}$	Point C3

Example Response

D	For indicating $x_{\text{new}} < x_2$	Point D1
	For a correct justification, such as:	Point D2
	- Comparing the change in momentum of either piece in the two cases, the speed of the corresponding piece after the projectile breaks apart, and the distance traveled before reaching the ground - Relating the motion of the center of mass of the projectile to the mass of the pieces and the distance each piece lands from the center of mass	

Example Responses

In the case where Piece Q falls straight to the ground, it has zero momentum, as opposed to a negative momentum when it moved backwards. Since momentum is conserved when the pieces break apart, Piece R must have less positive momentum in the new case. Since Piece R has less momentum it has a lower speed. It takes the same time to fall to the ground in both cases, so Piece R travels a shorter distance before landing in the new case.

OR

In the case where Piece Q falls straight to the ground, Piece Q lands closer to the center of mass of the system than when it moved backward. Since the smaller mass piece lands closer to the CM, the larger mass piece also must land closer to the CM than it did before. This means Piece R travels a shorter distance before landing in the second case.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

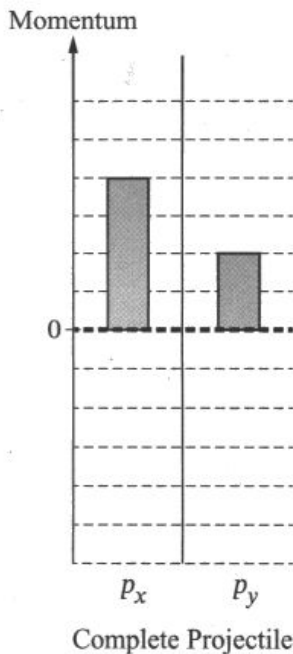
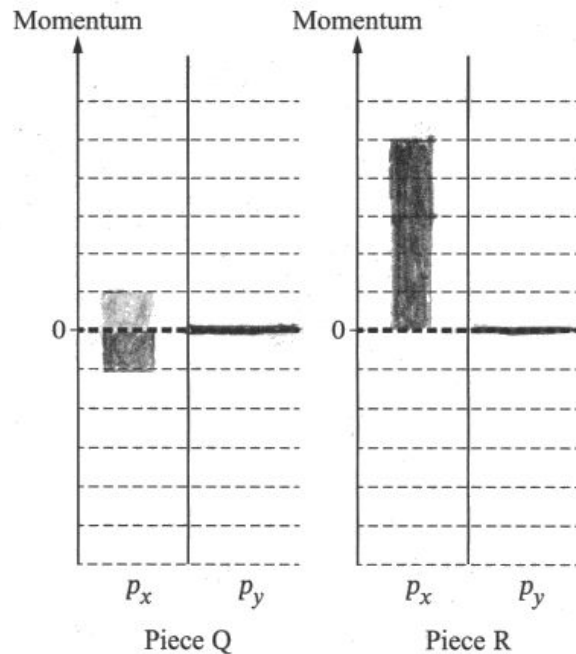


Figure 3

$t = t_1$



PART B

$$P_i = P_f$$

$$m_i v_i = m_f v_f$$

$$4M V_0 \cos \theta = 3M V_f + M(-V_0 \cos \theta)$$

$$4V_0 \cos \theta = 3V_f + V_0 \cos \theta$$

$$5V_0 \cos \theta = 3V_f$$

$$V_f = \frac{5}{3} V_0 \cos \theta$$

$$V_y = V_{y0} + a_y t$$

$$0 = V_0 \sin \theta + (-g)t$$

$$gt = V_0 \sin \theta$$

$$t = \frac{V_0 \sin \theta}{g}$$

$$\Delta x_1 = V_0 \cos \theta \cdot \frac{V_0 \sin \theta}{g}$$

$$= \frac{V_0^2 \sin \theta \cos \theta}{g}$$

$$x_2 = \frac{V_0^2 \sin \theta \cos \theta}{g} + \frac{5V_0^2 \sin \theta \cos \theta}{3g}$$

$$\Delta x_2 = V_f \left(\frac{V_0 \sin \theta}{g} \right)$$

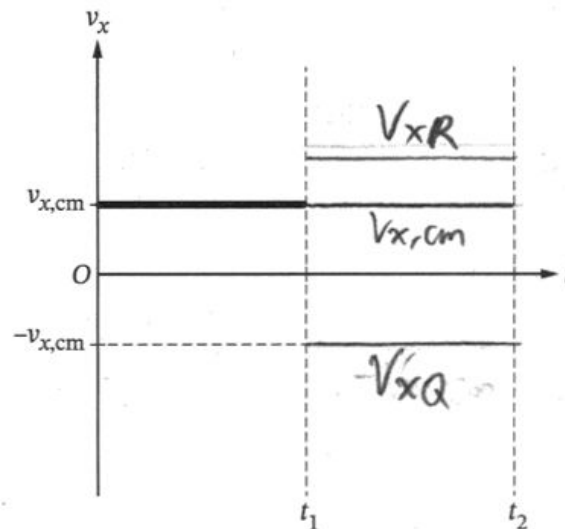
$$\Delta x_2 = \frac{5V_0^2 \sin \theta \cos \theta}{3g}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D $x_{\text{new}} < x_2$, since momentum is conserved before and after the separation, the velocity of the center of mass has to be constant throughout the process. In the previous scenario, part of the block is going to the left after separation, requiring more velocity on piece R to cancel out velocity to the left than when piece Q is stationary. Since the velocity on piece R is less, and airborne time is constant, x_{new} would be shorter than x_2 .

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

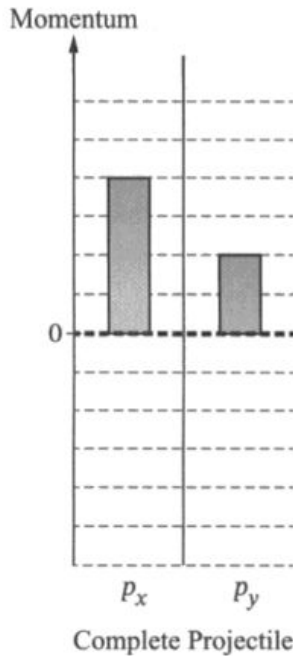
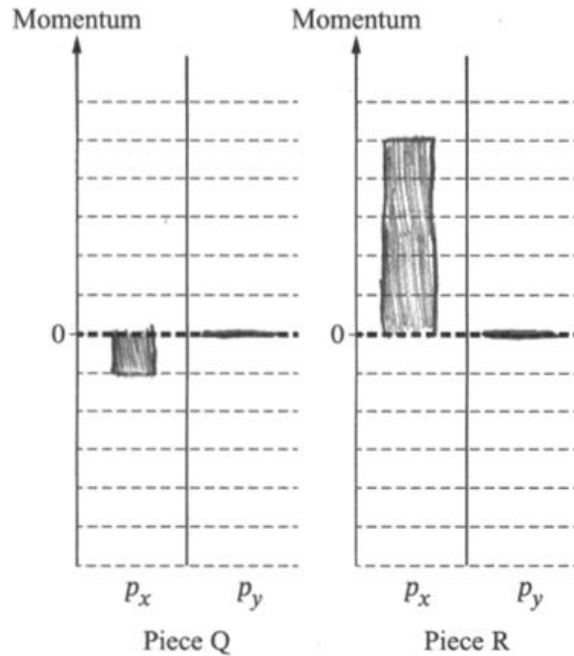


Figure 3

$t = t_1$



PART B

horizontal momentum is conserved

just before t_1 : $\sum p_x = 4Mv_0 \cos \theta$

just after t_1 : $\sum p_x = 3Mv_R - Mv_A$

$$4Mv_0 \cos \theta = 3Mv_R - Mv_A$$

$$4v_0 \cos \theta = 3v_R - v_A$$

since A falls to $x=0$,

$$v_A = v_0 \cos \theta$$

$$4v_0 \cos \theta = 3v_R - v_0 \cos \theta$$

$$3v_R = 5v_0 \cos \theta$$

$$v_R = \frac{5}{3}v_0 \cos \theta$$

$$v_y = v_{y0} + at$$

$$0 = v_0 \sin \theta - g t_{\text{top}}$$

$$t_{\text{top}} = \frac{v_0 \sin \theta}{g}$$

Δx at top:

$$\Delta x = v_x t$$

$$\Delta x = v_0 \cos \theta \left(\frac{v_0 \sin \theta}{g} \right)$$

$$\Delta x_{\text{top}} = \frac{v_0^2 \cos \theta \sin \theta}{g}$$

$$\Delta x_{\text{bottom}} = v_R t_{\text{bottom}}$$

$$= \frac{5}{3} v_0 \cos \theta \left(\frac{v_0 \sin \theta}{g} \right)$$

$$= \frac{5}{3} \left(\frac{v_0^2 \cos \theta \sin \theta}{g} \right)$$

$$x_2 = \Delta x_{\text{top}} + \Delta x_{\text{bottom}}$$

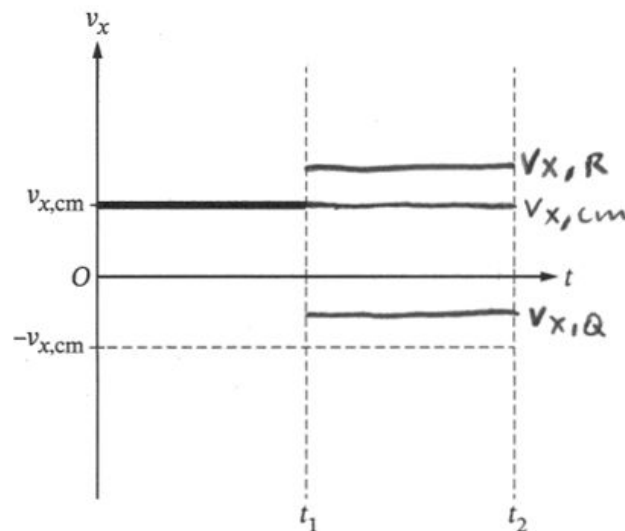
$$x_2 = \frac{8}{3} \left(\frac{v_0^2 \cos \theta \sin \theta}{g} \right)$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$x_{\text{new}} < x_2$. This is because during the explosion, momentum is conserved, in the horizontal direction. Thus, the momentum that the complete piece had in the horizontal direction must be equal to the sum of the horizontal momenta of pieces R and Q. Since Q falls straight down, all of the horizontal momentum is transferred to R in this case. Therefore, since $p = mv$, and the mass of R is the same as in the first scenario, v must be greater. Since R has a greater horizontal velocity, it will travel farther since $\text{Range} = v_{x0}t$. It is the same since the vertical motion is free fall in both scenarios.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

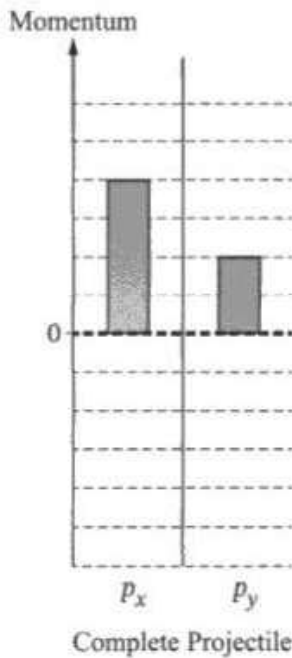
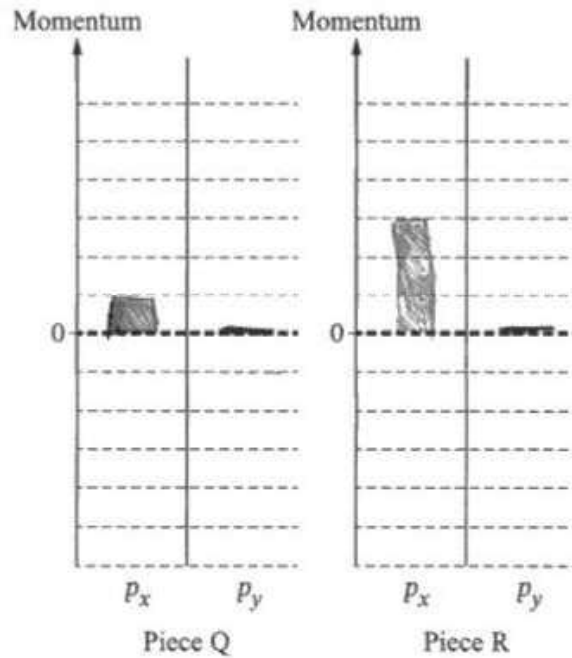


Figure 3

$t = t_1$



PART B

$$x: p = mv$$

$$y: 0 = v_i \sin \theta - gt_1$$

$$4mv_i \cos \theta = (3m)(v_R) \cos \theta - mv_i \cos \theta \quad t_1 = \frac{v_i \sin \theta}{g}$$

$$v_R = \frac{5mv_i \cos \theta}{3m}$$

$$v_R = \frac{5v_i \cos \theta}{3}$$

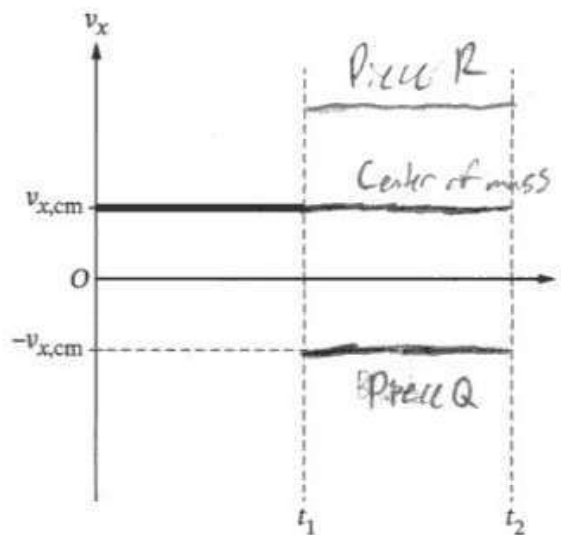
$$X = \left(\frac{5v_i \cos \theta}{3} \right) \left(\frac{v_i \sin \theta}{g} \right) = \frac{5v_i^2 \cos \theta \sin \theta}{3g}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$$x_{\text{new}} < x_2$$

If piece Q falls straight down, that means the change in momentum was less for piece Q. This means the change in momentum for piece R must be smaller as well, resulting in a smaller velocity and smaller distance traveled.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

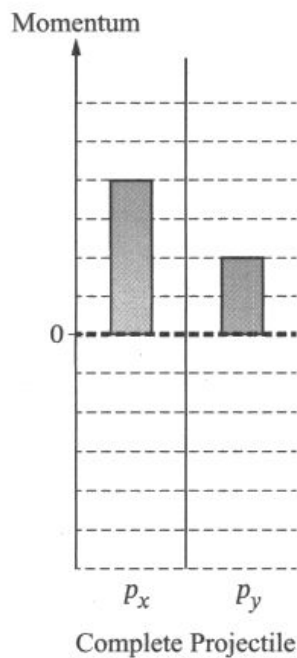
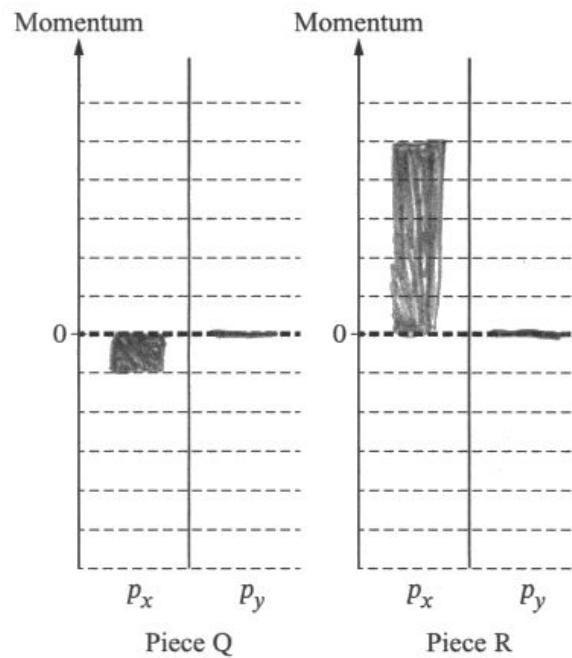


Figure 3

$t = t_1$



PART B

$$p = mv, \quad p_{x_i} = 4M v_x = 4M v_0 \cos \theta$$

$$\Delta x = Vt, \quad V_f = V_i + at$$

$$0 = v_0 \sin \theta + g t_1 \rightarrow t_1 = \frac{v_0 \sin \theta}{g} \rightarrow t_1 = \frac{1}{2} t_2$$

$$x_2 = v_0 \cos \theta \left(\frac{v_0 \sin \theta}{g} \right) + \frac{5}{4} v_0 \cos \theta \left(\frac{v_0 \sin \theta}{g} \right) = \frac{9}{4} \left(\frac{v_0^2 \cos \theta \sin \theta}{g} \right)$$

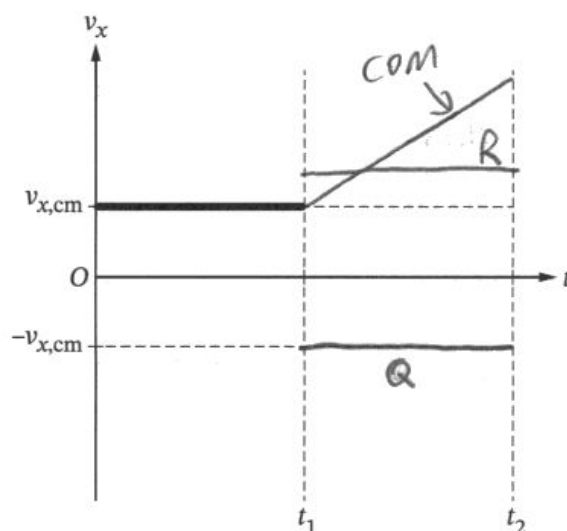
$$4M v_0 \cos \theta = -M v_0 \cos \theta + 5M v_0 \cos \theta$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$$x_{\text{new}} < x_2.$$

Due to conservation of momentum between the two blocks/the system, Velocity of piece R is less in the new scenario, after breaking apart.

Piece Q had negative momentum originally, but now has zero momentum after breaking apart, so piece R has less momentum, so less velocity, less distance traveled.

$$p_i = p_f.$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

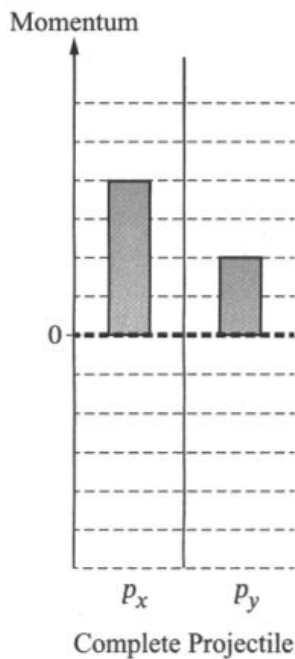
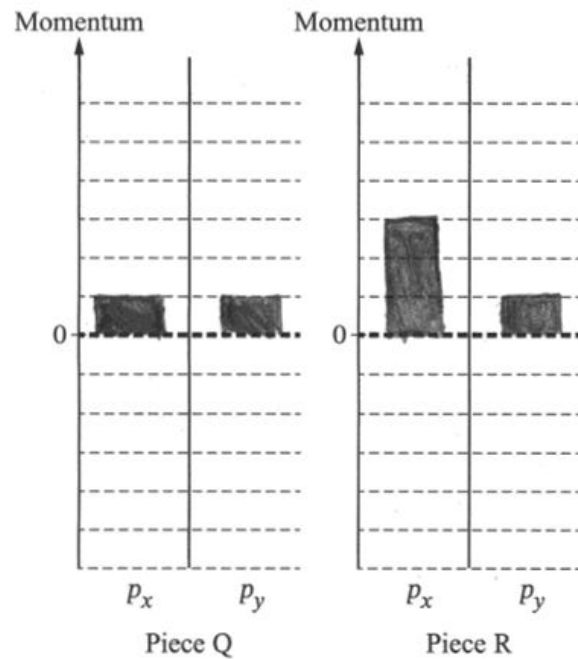


Figure 3

$t = t_1$



PART B

$$\frac{x_2}{2} = v_0 \cos(\theta) \left(\frac{-2v_0 \sin(\theta)}{g} \right)$$

$$x_2 = 2v_0 \cos(\theta) \left(\frac{-2v_0 \sin(\theta)}{g} \right)$$

$$\Delta x = v_{x0}t + \frac{1}{2}a_x t^2$$

$$0 = v_0 \sin(\theta)t + \frac{1}{2}gt^2$$

$$-v_0 \sin(\theta)t = \frac{1}{2}gt^2$$

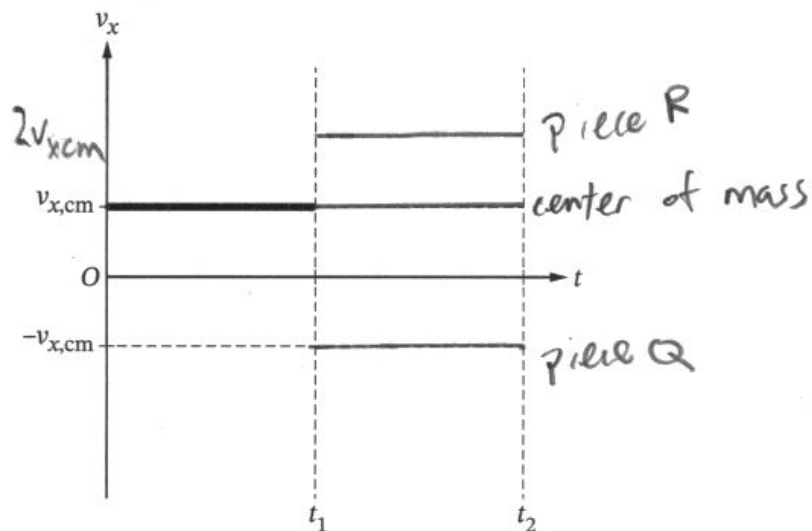
$$\frac{-2v_0 \sin(\theta)}{g} = t$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$x_{\text{new}} < x_2$. we know this because the center of mass, as shown in figure 4, is conserved at $v_{x,cm}$. If piece q falls straight down, piece r can't travel as far right as x_2 because it would cause the center of mass to be too far to the right compared to if they didn't separate. Thus to make sure the center of mass stays constant.

Go to Question 3 when you're done with this question.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

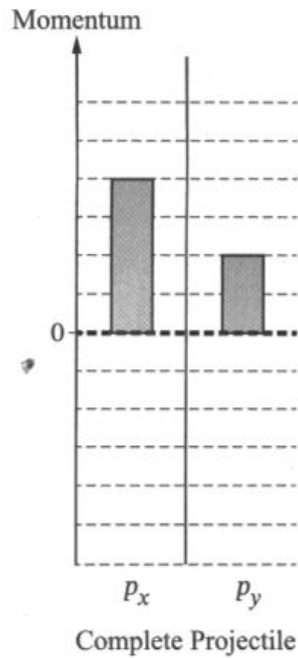
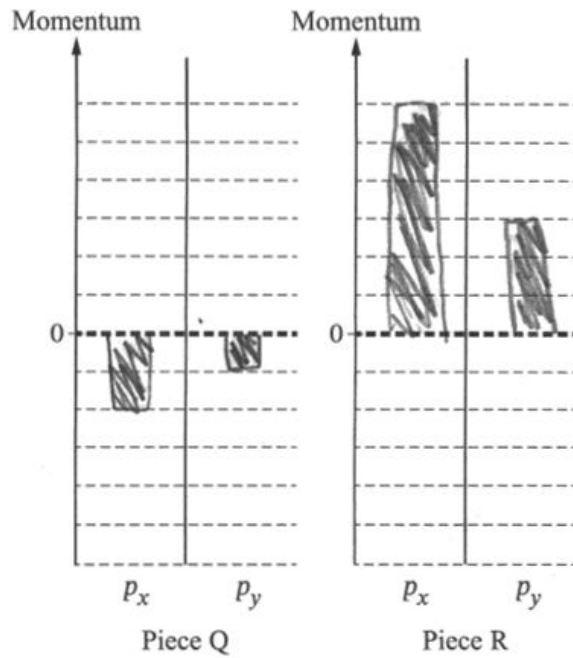


Figure 3

$t = t_1$



PART B

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$0 = v_0^2 + 2(10)(x_2 - 0)$$

$$v_0^2 = -20x_2$$

$$p = mv$$

$$\cos \theta = \frac{x}{v_0}$$

$$4m v_0 \cos \theta = 3m v_R - m v_Q$$

$$\frac{4m v_0 \cos \theta + m v_Q}{3m} = v_R$$

$$600 = 30x + 10y$$

$$60 = 3x + y$$

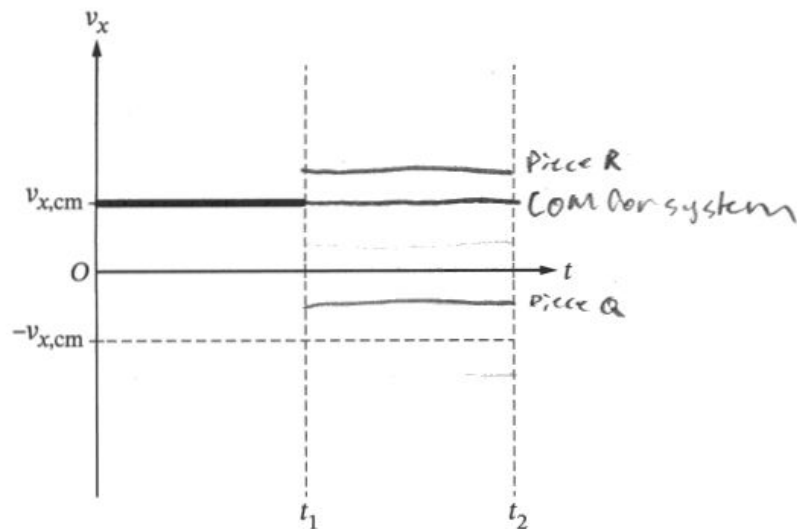
$$y = 60 - 3x$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$$x_{\text{new}} < x_2$$

this is because if Q does not go backwards then R will not need as great of a speed to conserve momentum. As the speed will be less, it will go less far. Referencing part A, if Q was at 0 then the momentum for R would be less.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

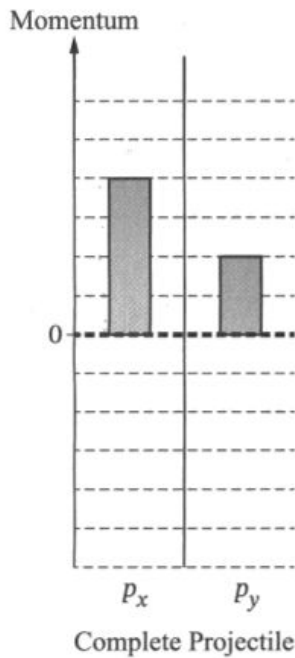
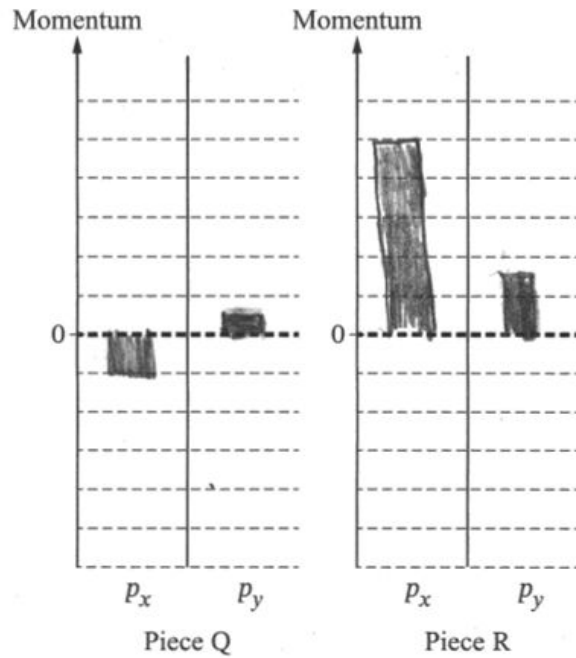


Figure 3

$t = t_1$



PART B

$$F = ma$$

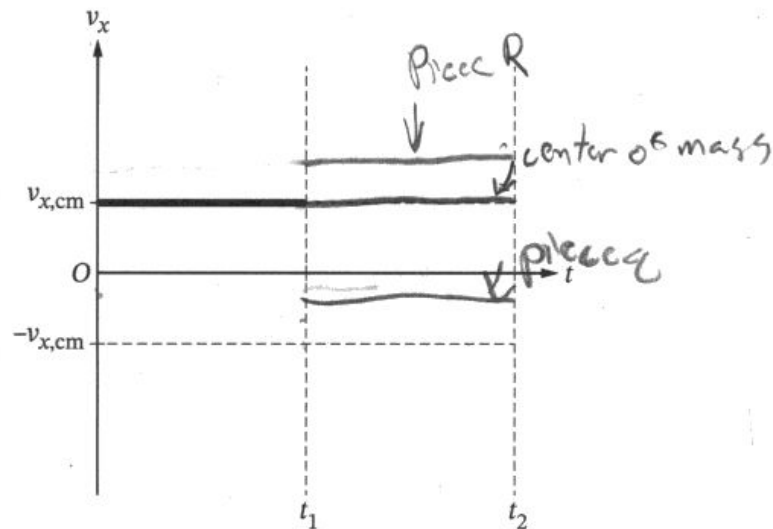
$$\Delta x = \int v_x(t) dt$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$x_{\text{new}} < x_2$ Since the velocity of piece Q is no longer negative it does not have negative momentum so piece R does not have to make up for it so it will have less v_x therefore it will go less distance.

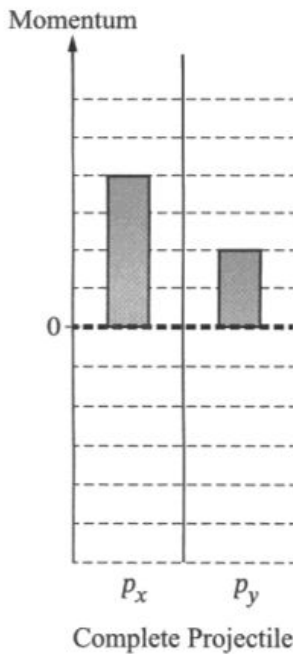
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

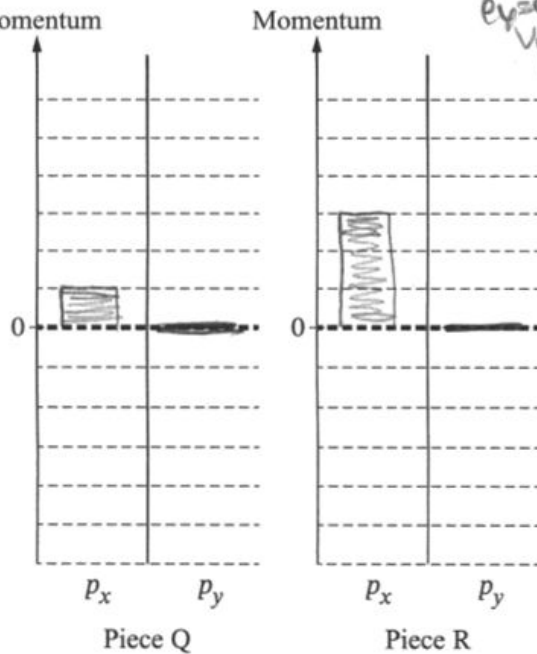
$t = 0$



p_x constant according
to m
b/c
 v_x constant
add to
same

Figure 3

$t = t_1$



$p_y = 0$
 $v_y = 0$ at
highest
point

PART B

$$v_{yx} = v_{y0} + at$$

$$v_{yx} = 0 \text{ at max height}$$

$$0 = v_0 \sin \theta - gt$$

$$gt = v_0 \sin \theta$$

$$t = \frac{v_0 \sin \theta}{g}$$

$$x_2 = x_0 + v_{0x}t + \frac{1}{2}at^2 \quad a_x = 0$$

$$x_2 = x_0 + v_{0x}t \quad x_0 = 0$$

$$x_2 = 0 + v_{0x}t$$

$$x_2 = v_0 \cos \theta t$$

$$x_2 = v_0 \cos \theta \cdot \frac{v_0 \sin \theta}{g}$$

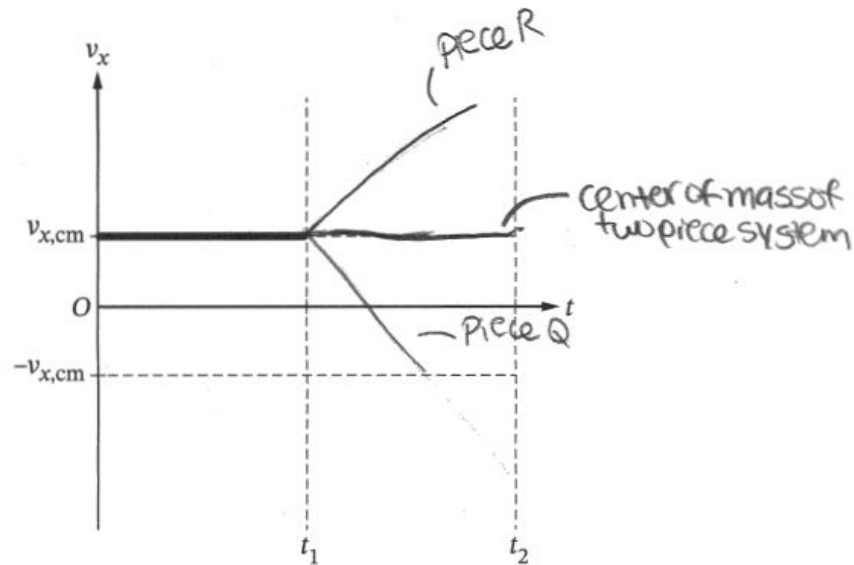
$$x_2 = \frac{v_0^2 \cos \theta \sin \theta}{g}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$$x_{\text{new}} > x_2$$

In the second case where piece Q falls straight down, more velocity is transferred to piece R as piece Q does not have any more displacement in the x. Since momentum is conserved for the Q&R block system, if piece Q does not have any x displacement and thus no x component of velocity, more velocity can be transferred to piece R. Since piece R has greater overall velocity, both x & y components, it will travel farther.

*than before when piece Q did not fall straight down & had an x component of velocity

Go to Question 3 when you're done with this question.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

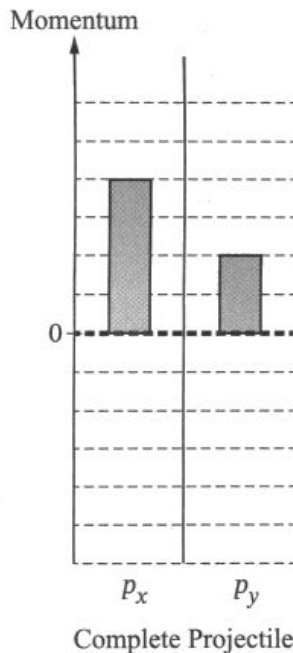
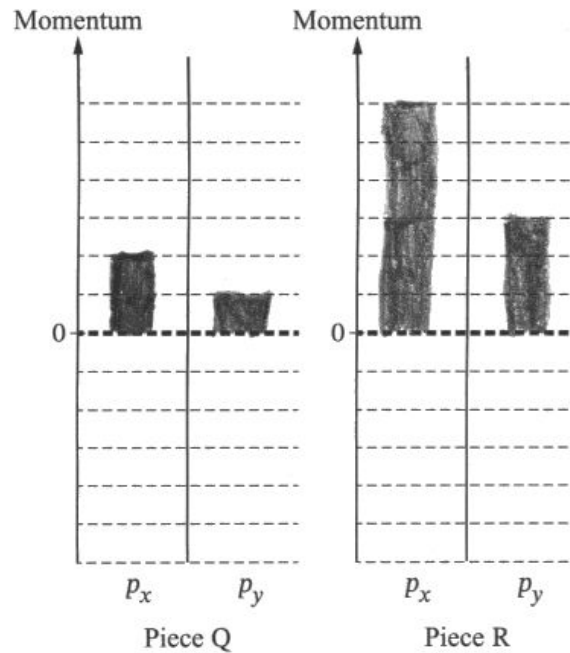


Figure 3

$t = t_1$



PART B

$X_2 = ?$ $V_0 \theta$

$P_i = P_f$
 $4mv_0 = 3mv_1 + mv_2$
 $4mv_0 \cos \theta = 3mv_1 \cos \theta + mv_2 \cos \theta$
 $t_2 V_0 \cos \theta = X_2$
 $V_x = \frac{\Delta X_2}{t_2}$
 $V_x = V_0 \cos \theta$
 $X_2 = \frac{V_0 \sin \theta - V_0 \cos \theta \cdot V_0 \cos \theta}{-g}$
 $t_2 = \frac{V_0 \sin \theta - V_0 \cos \theta}{-g}$

X-D	Y-D
$V_x = V_0 \cos \theta$	$V_{yi} = V_0 \sin \theta$
$\Delta X = X_2$	$V_{yf} =$
	$a = -g$
	$\Delta Y =$

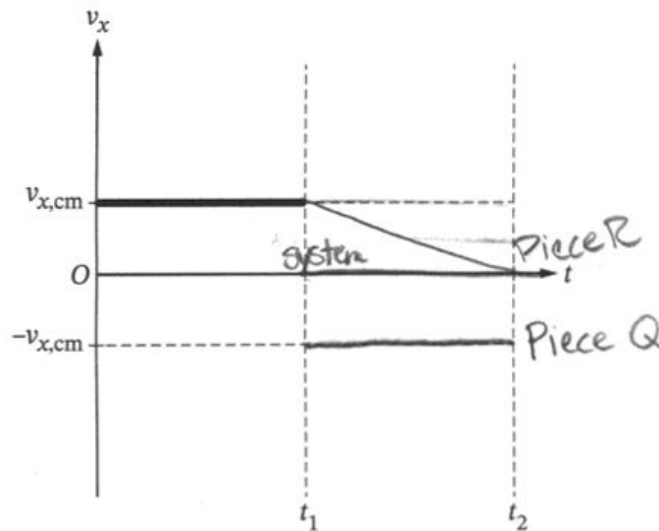
$t = t_2$
 $V_{yf} = V_i + a t_2$
 $\frac{V_{yf} - V_{yi}}{a} = t_2$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$$X_{\text{new}} > X_2$$

because $p_i = p_f$,
 so with less momentum
 in the x -direction for Piece
 Q, Piece R must have a larger
 velocity in the x -direction,
 carrying it a farther distance.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART A

Figure 2

$t = 0$

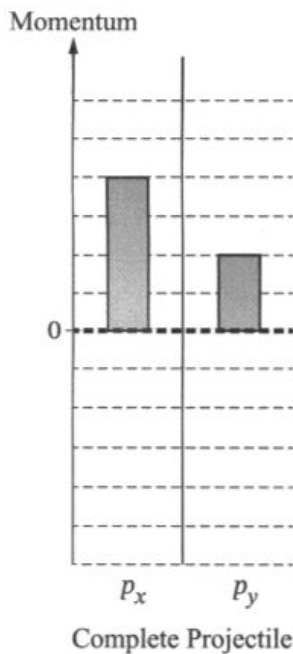
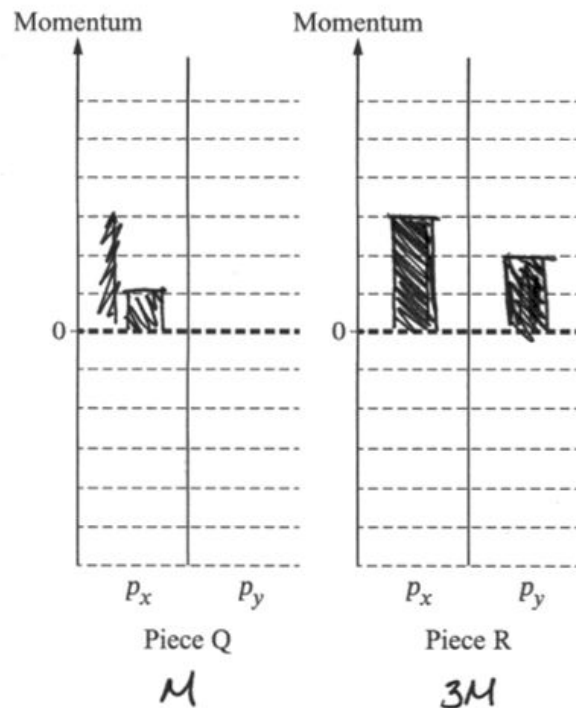


Figure 3

$t = t_1$



PART B

	x_R	y_R
mass _o	$4M$	$4M$
mass _f	$3M$	$3M$
vel _o	X	V_0

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

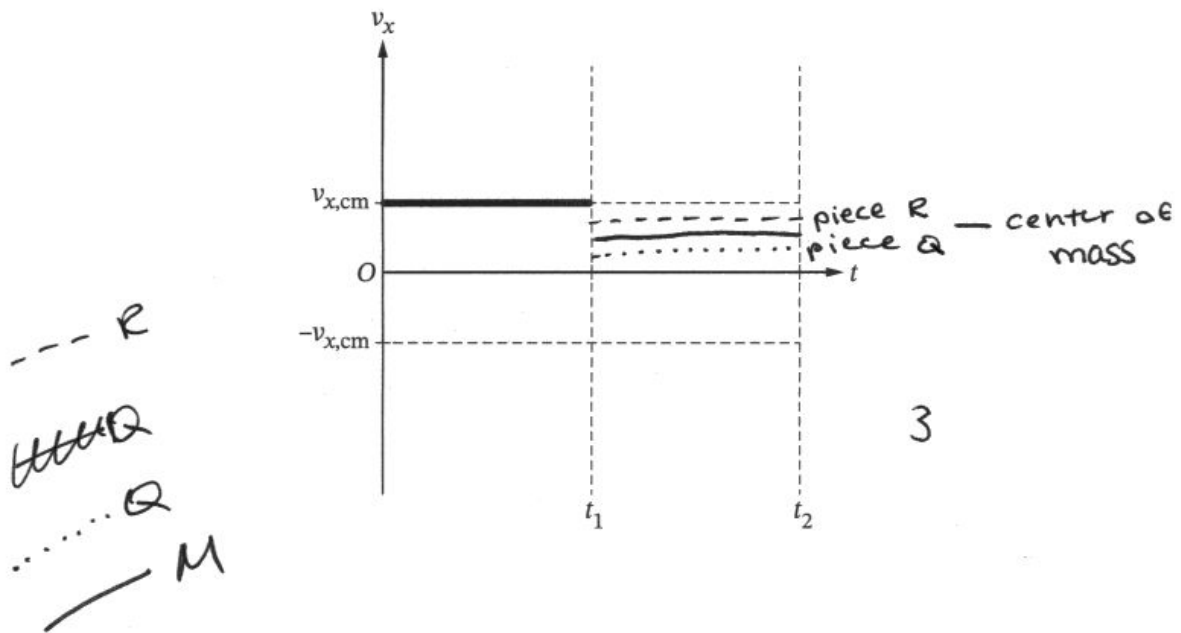
$$x_2 = 0 + v_{x0}t + \frac{1}{2} \cdot 0$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2: Version J

PART C

Figure 4



PART D

$x_{\text{new}} = x_2$ because conservation of momentum, $mv = m_1v_1 + m_2v_2$ & $m_1v_1 = 0$
 so it will still land in the same place because the masses are still the same after the break, and piece P still lands @ at the initial position & only moves vertically

Translating Between Representations (TBR)

Sample: 2A

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 1

TBR Part A Point 3 Score: 1

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 1

TBR Part B Point 3 Score: 1

TBR Part B Point 4 Score: 1

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 1

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 1

Score: 12

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units.
- Point A2 was earned. The response indicates that both Pieces Q and R have zero vertical momentum.
- Point A3 was earned. The response indicates the horizontal momentum amounts of negative one unit for Piece Q and positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes conservation of momentum.
- Point B2 was earned. The response indicates that the vertical component of the projectile's velocity is zero at time t_1 , as shown by the method used to determine t_1 .
- Point B3 was earned. The response correctly uses conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was earned. The response includes a correct expression for x_2 . The expression shown is equivalent to the final answer point shown in the example response.

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $u_{x,cm}$.
- Point C2 was earned. The response indicates a horizontal line for Piece R that is greater than $u_{x,cm}$.

- Point C3 was earned. The response indicates a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was earned. The response indicates that the motion of the center of mass does not change and since the velocity of Piece Q changes from negative to zero in the new scenario, the velocity of Piece R must be less and therefore it travels a shorter distance than in the original scenario.

Sample: 2B

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 1

TBR Part A Point 3 Score: 1

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 1

TBR Part B Point 3 Score: 1

TBR Part B Point 4 Score: 1

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 0

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 0

Score: 10

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units.
- Point A2 was earned. The response indicates that both Pieces Q and R have zero vertical momentum.
- Point A3 was earned. The response indicates the horizontal momentum amounts of negative one unit for Piece Q and positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations.
- Point B2 was earned. The response indicates that the vertical component of the projectile's velocity is zero at time t_1 , as shown in the expression obtained for t_{top} .
- Point B3 was earned. The response correctly uses conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was earned. The response includes a correct expression for x_2 .

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $u_{x,cm}$.
- Point C2 was earned. The response indicates a horizontal line for Piece R that is greater than $u_{x,cm}$.
- Point C3 was not earned. The response does not indicate a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was not earned. The response does not indicate that due to the conservation of momentum that Piece R has less momentum in the new scenario and therefore it travels a shorter distance than in the original scenario.

Sample: 2C

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 1

TBR Part A Point 3 Score: 0

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 1

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 1

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 1

Score: 9

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units. The response shows Piece Q with positive one unit and Piece R with four units.
- Point A2 was earned. The response indicates that both Pieces Q and R have zero vertical momentum.
- Point A3 was not earned. The response does not indicate the horizontal momentum amounts of negative one unit for Piece Q or positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations.
- Point B2 was earned. The response indicates that the vertical component of the projectile's velocity is zero at time t_1 , as shown by the method used to determine t_1 .
- Point B3 was not earned. The response does not use conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 . The response shows an equation that cannot be solved to arrive at the correct answer for v_R as the term $\cos \theta$ should not appear in the momentum expression for Piece R.
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $v_{x,cm}$.
- Point C2 was earned. The response indicates a horizontal line for Piece R that is greater than $v_{x,cm}$.

- Point C3 was earned. The response indicates a horizontal line for Piece Q that is equal to negative $v_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was earned. The response indicates that the change in momentum of Piece R is smaller in the new situation and therefore it travels less distance than the original situation.

Sample: 2D

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 1

TBR Part A Point 3 Score: 1

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 1

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 0

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 1

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 1

Score: 9

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units.
- Point A2 was earned. The response indicates that both Pieces Q and R have zero vertical momentum.
- Point A3 was earned. The response indicates horizontal momentum amounts of negative one unit for Piece Q and positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations.
- Point B2 was earned. The response indicates that the vertical component of the projectile's velocity is zero at time t_1 .
- Point B3 was not earned. The response does not correctly use conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 as Piece R is shown as mass $5M$ in the response.
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was not earned. The response does not indicate a horizontal line for the center of mass that is equal to $u_{x,cm}$.
- Point C2 was earned. The response does indicate a horizontal line for Piece R that is greater than $u_{x,cm}$.
- Point C3 was earned. The response indicates a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was earned. The response indicates that due to the conservation of momentum that Piece R has less momentum in the new scenario and so therefore it travels a shorter distance than in the original scenario.

Sample: 2E

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 0

TBR Part A Point 3 Score: 0

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 1

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 1

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 1

Score: 8

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units.
- Point A2 was not earned. The response does not indicate that both Pieces Q and R have zero vertical momentum.
- Point A3 was not earned. The response does not indicate the horizontal momentum amounts of negative one unit for Piece Q or positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations.
- Point B2 was earned. The response indicates that the vertical displacement is zero at time t_2 . Although the original equation of motion is shown with horizontal components, values consistent with the vertical displacement are shown directly below it.
- Point B3 was not earned. The response does not correctly use conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $v_{x,cm}$.
- Point C2 was earned. The response indicates a horizontal line for Piece R that is greater than $v_{x,cm}$.
- Point C3 was earned. The response indicates a horizontal line for Piece Q that is equal to negative $v_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was earned. The response indicates that the location of the center of mass does not change and since the location Piece Q lands is closer to location of the center of mass in the new situation, the location Piece R lands must be less as well.

Sample: 2F

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 0

TBR Part A Point 3 Score: 0

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 0

TBR Part B Point 3 Score: 1

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 0

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 1

Score: 7

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units. Piece Q is shown as negative two units, and Piece R is shown as six units.
- Point A2 was not earned. The response does not indicate that both Pieces Q and R have zero vertical momentum.
- Point A3 was not earned. The response does not indicate the horizontal momentum amounts of negative one unit for Piece Q or positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations.
- Point B2 was not earned. The response does not indicate that the vertical component of the projectile's velocity is zero at time t_1 , or that the vertical displacement is zero at time t_2 . The response shows velocity set to zero, but it is unclear that it is a vertical velocity.
- Point B3 was earned. The response indicates a correct conservation of momentum equation that can be used to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $v_{x,cm}$.
- Point C2 was earned. The response indicates a horizontal line for Piece R that is greater than $v_{x,cm}$.

- Point C3 was not earned. The response does not indicate a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was earned. The response indicates that the change in momentum of Piece R is less in the new scenario and therefore travels a shorter distance than in the original scenario.

Sample: 2G

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 0

TBR Part A Point 3 Score: 1

TBR Part B Point 1 Score: 0

TBR Part B Point 2 Score: 0

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 1

TBR Part C Point 3 Score: 0

TBR Part D Point 1 Score: 1

TBR Part D Point 2 Score: 1

Score: 6

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units.
- Point A2 was not earned. The response does not indicate that both Pieces Q and R have zero vertical momentum.
- Point A3 was earned. The response indicates the horizontal momentum amounts of negative one unit for Piece Q and positive five units for Piece R.

B.

- Point B1 was not earned. The response does not include a multistep derivation that includes kinematics equations.
- Point B2 was not earned. The response does not indicate that the vertical component of the projectile's velocity is zero at time t_1 , or that the vertical displacement is zero at time t_2 .
- Point B3 was not earned. The response does not correctly use conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $u_{x,cm}$.
- Point C2 was earned. The response indicates a horizontal line for Piece R that is greater than $u_{x,cm}$.
- Point C3 was not earned. The response does not indicate a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was earned. The response indicates that x_{new} is less than x_2 .
- Point D2 was earned. The response indicates that the change in momentum of Piece R is greater in the original scenario and therefore it has less horizontal velocity in the new situation and therefore travels less distance.

Sample: 2H

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 1

TBR Part A Point 3 Score: 0

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 1

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 1

TBR Part C Point 2 Score: 0

TBR Part C Point 3 Score: 0

TBR Part D Point 1 Score: 0

TBR Part D Point 2 Score: 0

Score: 5

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units. Piece Q is shown as one unit and Piece R is shown as three units.
- Point A2 was earned. The response indicates that both Pieces Q and R have zero vertical momentum.
- Point A3 was not earned. The response does not indicate the horizontal momentum amounts of negative one unit for Piece Q or positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations.
- Point B2 was earned. The response indicates that the vertical component of the projectile's velocity is zero at time t_1 .
- Point B3 was not earned. The response does not correctly use conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was earned. The response indicates a horizontal line for the center of mass that is equal to $v_{x,cm}$.
- Point C2 was not earned. The response does not indicate a horizontal line for Piece R that is greater than $v_{x,cm}$.
- Point C3 was not earned. The response does not indicate a horizontal line for Piece Q that is equal to negative $v_{x,cm}$.

D

- Point D1 was not earned. The response does not indicate that x_{new} is less than x_2 .
- Point D2 was not earned. The response does not indicate that the change in momentum of Piece R is less in the new scenario and therefore it travels a shorter distance than in the original scenario.

Sample: 2I

TBR Part A Point 1 Score: 0

TBR Part A Point 2 Score: 0

TBR Part A Point 3 Score: 0

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 0

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 0

TBR Part C Point 2 Score: 0

TBR Part C Point 3 Score: 1

TBR Part D Point 1 Score: 0

TBR Part D Point 2 Score: 0

Score: 2

A.

- Point A1 was not earned. The response does not include bars indicating horizontal momentum of Pieces Q and R that add up to four units.
- Point A2 was not earned. The response does not indicate that both Pieces Q and R have zero vertical momentum.
- Point A3 was not earned. The response does not indicate the horizontal momentum amounts of negative one unit for Piece Q or positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes momentum equations.
- Point B2 was not earned. The response does not indicate that the vertical component of the projectile's velocity is zero at time t_1 , or that the vertical displacement is zero at time t_2 .
- Point B3 was not earned. The response includes a conservation of momentum equation for the horizontal component of velocity for Piece R after t_1 , but the horizontal velocities are not correctly indicated in the equation.
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was not earned. The response does not indicate a horizontal line for the center of mass that is equal to $u_{x,cm}$.
- Point C2 was not earned. The response does not indicate a horizontal line for Piece R that is greater than $u_{x,cm}$.
- Point C3 was earned. The response indicates a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was not earned. The response does not indicate that x_{new} is less than x_2 .
- Point D2 was not earned. The response does not indicate that the change in momentum of Piece R is less in the new scenario and therefore travels a shorter distance than in the original scenario.

Sample: 2J

TBR Part A Point 1 Score: 1

TBR Part A Point 2 Score: 0

TBR Part A Point 3 Score: 0

TBR Part B Point 1 Score: 1

TBR Part B Point 2 Score: 0

TBR Part B Point 3 Score: 0

TBR Part B Point 4 Score: 0

TBR Part C Point 1 Score: 0

TBR Part C Point 2 Score: 0

TBR Part C Point 3 Score: 0

TBR Part D Point 1 Score: 0

TBR Part D Point 2 Score: 0

Score: 2

A.

- Point A1 was earned. The response includes bars indicating horizontal momentum of Pieces Q and R that add up to four units. Piece M is positive one unit and Piece R is positive three units.
- Point A2 was not earned. The response does not indicate that both Pieces Q and R have zero vertical momentum.
- Point A3 was not earned. The response does not indicate the horizontal momentum amounts of negative one unit for Piece Q or positive five units for Piece R.

B.

- Point B1 was earned. The response includes a multistep derivation that includes kinematics equations. An equation of motion is shown with more than one significant change made to the equation.
- Point B2 was not earned. The response does not indicate that the vertical component of the projectile's velocity is zero at time t_1 , or that the vertical displacement is zero at time t_2 .
- Point B3 was not earned. The response does not correctly use conservation of momentum to determine the horizontal component of velocity for Piece R after t_1 .
- Point B4 was not earned. The response does not include a correct expression for x_2 .

C

- Point C1 was not earned. The response does not indicate a horizontal line for the center of mass that is equal to $v_{x,cm}$.
- Point C2 was not earned. The response does not indicate a horizontal line for Piece R that is greater than $v_{x,cm}$ shown before t_1 .

- Point C3 was not earned. The response does not indicate a horizontal line for Piece Q that is equal to negative $u_{x,cm}$.

D

- Point D1 was not earned. The response does not indicate that x_{new} is less than x_2 .
- Point D2 was not earned. The response does not indicate that the change in momentum of Piece R is less in the new scenario.