
AP[®] Physics C: Mechanics

Sample Student Responses and Scoring Commentary

Question #1

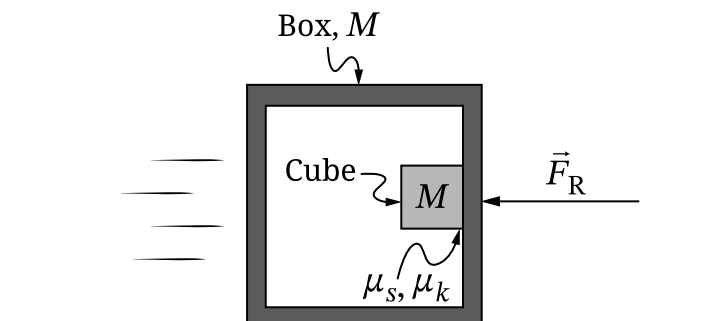
DRAFT

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Question 1: Version J

1. A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M , is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k , respectively, where $\mu_s > \mu_k$.

Figure 1

The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

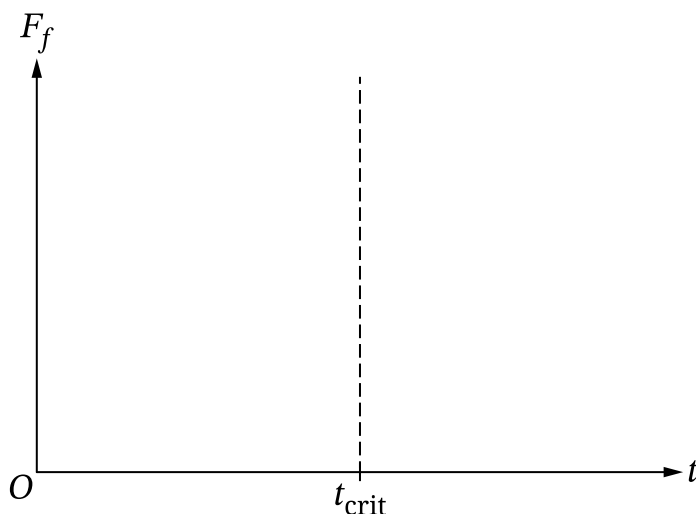
Consider the motion of the box and cube for the following times.

- At time $t = 0$, the box-cube system slides to the right with speed v_0 .
- For $0 \leq t < t_{\text{crit}}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box.
- At $t = t_{\text{crit}}$, the cube begins to slide downward along the wall of the box.

Part A

The magnitude of the normal force exerted on the cube by the wall of the box is F_N .

- i. **Determine** an expression for the magnitude of the acceleration of the cube. Express your answer in terms of M , F_N , and physical constants, as appropriate.
- ii. **Derive** an expression for F_N as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of M , b , v_0 , t , and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.
- iii. On the axes shown in Figure 2, **sketch** a graph of the magnitude F_f of the frictional force exerted on the cube by the wall of the box as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box.

Figure 2**Part B**

Derive an expression for t_{crit} in terms of M , b , μ_s , v_0 , and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

Question 1 Mathematical Routines (MR)**10 points****General Scoring Notes****NR (No Response):** A score of NR is only assigned to responses that are blank.

- | | | |
|----------|---|-----------------|
| A | (i) For determining a correct expression for the magnitude of the horizontal acceleration of the cube | Point A1 |
|----------|---|-----------------|

Example Response

$$a_{\text{cube}} = \frac{F_N}{M}$$

- | | | |
|--|---|-----------------|
| | (ii) For a multistep derivation that includes Newton's second law | Point A2 |
|--|---|-----------------|

For indicating that the magnitude of the acceleration is $\frac{bv}{2M}$	Point A3
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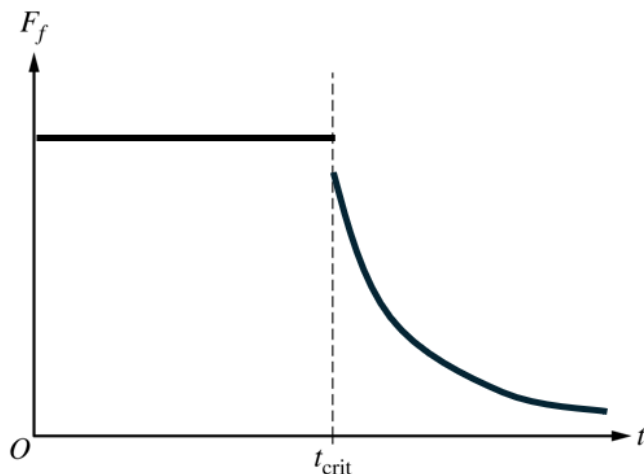
For separating dv and dt variables	Point A4
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For an exponential expression for F_N that evaluates to zero at infinite time	Point A5
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Example Response

$a_{\text{cube}} = a_{\text{box}} = a$ $a = \frac{F_{\text{net}}}{m_{\text{total}}} = -\frac{bv}{2M}$ $a = -\frac{bv}{2M}$	$a = \frac{dv}{dt} = -\frac{bv}{2M}$ $\frac{dv}{v} = -\frac{b}{2M} dt$ $\int_{v_0}^v \frac{1}{v} dv = -\frac{b}{2M} \int_0^t dt$ $\ln(v) - \ln(v_0) = -\frac{bt}{2M}$ $\ln\left(\frac{v}{v_0}\right) = -\frac{bt}{2M}$ $\frac{v}{v_0} = e^{-\frac{bt}{2M}}$ $v = v_0 e^{-\frac{bt}{2M}}$	$\vec{F}_N = Ma = M\left(-\frac{bv}{2M}\right)$ $\vec{F}_N = -\frac{b}{2}v_0 e^{-\frac{bt}{2M}}$ $F_N = \vec{F}_N = \frac{b}{2}v_0 e^{-\frac{bt}{2M}}$
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(iii)	For sketching a horizontal line from $t = 0$ to $t = t_{\text{crit}}$	Point A6
	For sketching a concave up curve that never increases for $t > t_{\text{crit}}$	Point A7
Scoring Note: This point can be earned regardless of the discontinuity at $t = t_{\text{crit}}$.		

Example Response

B	For a multistep derivation that includes Newton's second law	Point B1
	For indicating that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit}	Point B2
	For substituting the expression for F_N derived in part A into the expression for static friction	Point B3
Scoring Note: Responses that omit the subscript on μ_s may earn this point.		

Example Response

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = 0$$

$$F_f = F_g$$

$$\mu_s F_{N,\text{cube}} = F_g$$

$$\mu_s \left(\frac{b}{2} v_0 e^{-\frac{bt_{\text{crit}}}{2M}} \right) = Mg$$

$$e^{-\frac{bt_{\text{crit}}}{2M}} = \frac{2Mg}{\mu_s b v_0}$$

$$-\frac{bt_{\text{crit}}}{2M} = \ln \left(\frac{2Mg}{\mu_s b v_0} \right)$$

$$t_{\text{crit}} = -\frac{2M}{b} \ln \left(\frac{2Mg}{\mu_s b v_0} \right)$$

$$t_{\text{crit}} = \frac{2M}{b} \ln \left(\frac{\mu_s b v_0}{2Mg} \right)$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A

$$i. \sum F = Ma = F_N$$

$$a = \frac{F_N}{m}$$

$$ii. \sum F_{x_B} = ma = -F_R$$

$$2Ma = -bv$$

$$2M \frac{dv}{dt} = -bv$$

$$\int_{v_0}^v \frac{dv}{v} = \int_0^t \frac{-b}{2M} dt$$

$$\ln v \Big|_{v_0}^v = -\frac{b}{2M} t \Big|_0^t$$

$$\ln v - \ln v_0 = -\frac{b}{2M} t$$

$$\ln \frac{v}{v_0} = -\frac{b}{2M} t$$

$$\frac{v}{v_0} = e^{-\frac{b}{2M} t}$$

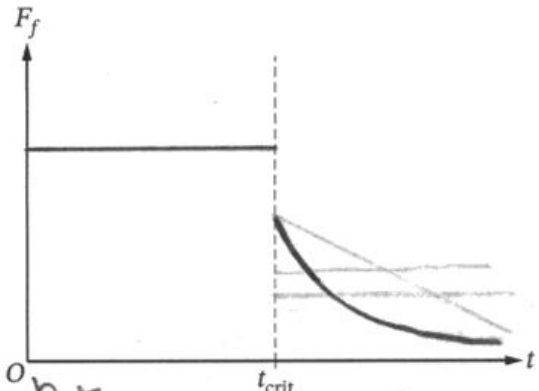
$$v = v_0 e^{-\frac{b}{2M} t}$$

$$a = \frac{d}{dt}(v)$$

$$a = \frac{d}{dt}(v_0 e^{-\frac{b}{2M} t})$$

$$a = -\frac{v_0 b}{2M} e^{-\frac{b}{2M} t}$$

Figure 2



$$\sum F_{xc} = ma = F_N$$

$$M \left(-\frac{v_0 b}{2M} e^{-\frac{b}{2M} t} \right) = F_N$$

$$-\frac{v_0 b}{2} e^{-\frac{b}{2M} t} = F_N$$

PART B

$$\sum F_{xc} = 0 = F_f - F_g$$

$$0 = \mu_s F_N - Mg$$

$$Mg = \mu_s \left(-\frac{v_0 b}{2} e^{-\frac{b}{2M} t_{crit}} \right)$$

$$\frac{Mg}{\mu_s} = -\frac{v_0 b}{2} e^{-\frac{b}{2M} t_{crit}}$$

$$-\frac{2Mg}{\mu_s v_0 b} = e^{-\frac{b}{2M} t_{crit}}$$

$$\ln \left(\frac{-2Mg}{\mu_s v_0 b} \right) = -\frac{b}{2M} t_{crit}$$

$$t_{crit} = -\frac{2M}{b} \ln \left(\frac{-2Mg}{\mu_s v_0 b} \right)$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A

i)  $\Sigma F = Ma$
 $F_N = Ma$
 $a = \frac{F_N}{M}$

ii) $\Sigma F = ma$

$$F_R = 2Ma$$

$$a = \frac{F_R}{2M} = -\frac{bv}{2M}$$

$$F_N = M\left(-\frac{bv}{2M}\right)$$

$$|F_N| = \frac{1}{2}bv = M\left(-\frac{dv}{dt}\right)$$

$$\int_{v_0}^v \frac{1}{\frac{1}{2}bv} dv = -\int_0^t \frac{1}{M} dt$$

$$\ln\left(\frac{1}{2}bv\right) - \ln\left(\frac{1}{2}bv_0\right) = -\frac{1}{M}t$$

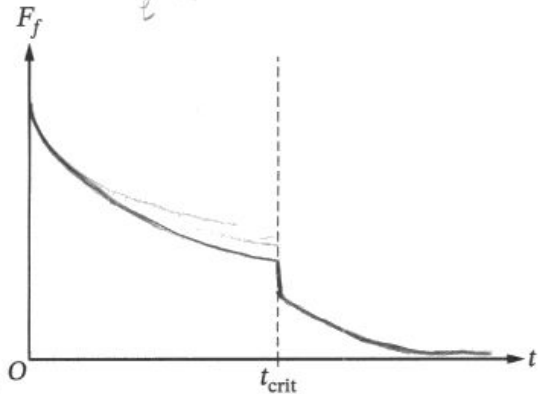
$$\ln\left(\frac{\frac{1}{2}bv}{\frac{1}{2}bv_0}\right) = -\frac{1}{M}t$$

$$\frac{\frac{1}{2}bv}{\frac{1}{2}bv_0} = e^{-\frac{1}{M}t}$$

$$\frac{1}{2}bv = \frac{1}{2}bv_0 e^{-\frac{1}{M}t}$$

$$F_N = \frac{1}{2}bv_0 e^{-\frac{1}{M}t}$$

Figure 2



PART B

$$\Sigma F_y = 0$$



$$\mu_s F_N = Mg$$

$$\frac{1}{2}\mu_s bv_0 e^{-\frac{1}{M}t_{crit}} = Mg$$

$$e^{-\frac{1}{M}t_{crit}} = \frac{2Mg}{\mu_s bv_0}$$

$$-\frac{1}{M}t_{crit} = \ln\left(\frac{2Mg}{\mu_s bv_0}\right)$$

$$t_{crit} = -M \cdot \ln\left(\frac{2Mg}{\mu_s bv_0}\right)$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A

$$\text{ii.) } \Sigma F = ma$$

$$-mg + F_{fk} = ma$$

$$-mg + \mu_k F_N = ma$$

$$a = \frac{-mg + \mu_k F_N}{m}$$

$$\text{iii.) } \Sigma F = ma$$

$$F_R = ma$$

$$-bv = m \frac{dv}{dt}$$

$$-\frac{dv}{v} = \frac{dt}{\tau}$$

$$-\int \frac{dv}{v} = \int \frac{dt}{\tau}$$

$$-\ln v = \frac{t}{\tau} + C$$

$$-\ln v + C = \frac{t}{\tau}$$

$$e^{-\ln v + C} = e^{\frac{t}{\tau}}$$

$$e^{-\ln v} e^C = e^{\frac{t}{\tau}}$$

$$\frac{1}{v} e^C = e^{\frac{t}{\tau}}$$

$$\frac{1}{v} = e^{\frac{t}{\tau} - C}$$

$$v = e^{C - \frac{t}{\tau}}$$

$$v(0) = v_0 = \frac{C}{b} e^{-0}$$

$$v_0 = \frac{C}{b} \Rightarrow C = bv_0$$

$$v(t) = v_0 e^{-\frac{bt}{m}}$$

$$a(t) = \frac{dv}{dt} = -\frac{b}{m} v_0 e^{-\frac{bt}{m}}$$

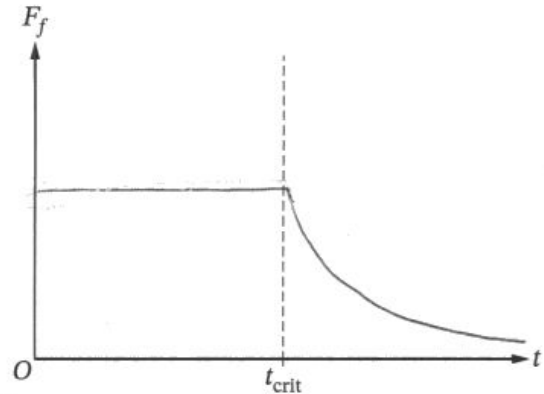
$$a(t) = -\frac{bv_0}{m} e^{-\frac{bt}{m}}$$

$$\Sigma F = ma$$

$$\Sigma F = m[a(t)]$$

$$F_N = m\left(-\frac{bv_0}{m} e^{-\frac{bt}{m}}\right) \rightarrow F_N = -bv_0 e^{-\frac{bt}{m}}$$

Figure 2



PART B

$$\Sigma F = ma$$

$$\Sigma F = 0$$

$$F_s - mg$$

$$\mu_s F_N = mg$$

$$\mu_s \left[-bv_0 e^{-\frac{bt}{m}} \right] = mg$$

$$e^{-\frac{bt}{m}} = \frac{mg}{\mu_s bv_0}$$

$$-\frac{bt}{m} = \ln\left(\frac{mg}{\mu_s bv_0}\right) \Rightarrow t_{crit} = -\frac{m}{b} \ln\left(\frac{mg}{\mu_s bv_0}\right)$$

$$-bt = \ln\left(\frac{mg}{\mu_s bv_0}\right)$$

$$t_{crit} = -\frac{m}{b} \ln\left(\frac{mg}{\mu_s bv_0}\right)$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

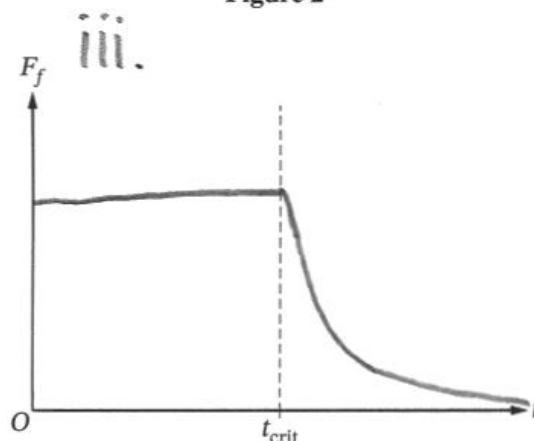
Question 1: Version J

PART A

i. $\Sigma F_x = M\vec{a} = -\vec{F}_R = -\vec{F}_N$
 $a = \frac{-F_N}{M}$ $|a| = \frac{F_N}{M}$

ii. $\Sigma F_x = M\vec{a} = -\vec{F}_N = -\vec{F}_R$
 $M \frac{dv}{dt} = -bv$
 $\int_{v_0}^v \frac{1}{v} dv = \int_0^t \frac{-b}{M} dt$
 $\ln|v| - \ln|v_0| = \frac{-b}{M} t$
 $\ln\left|\frac{v}{v_0}\right| = \frac{-bt}{M}$
 $v = v_0 e^{\frac{-bt}{M}}$
 $F_N = M \frac{dv}{dt} = M \frac{d}{dt} \left(v_0 e^{\frac{-bt}{M}} \right)$
 $F_N = -bv_0 e^{\frac{-bt}{M}}$

Figure 2



PART B

$\Sigma F_y = Ma_y = Mg - F_{kf}$ $F_{kf} = |M\vec{F}| = M_k b v_0 e^{\frac{-bt_{crit}}{M}}$

$Ma_y = Mg - M_k b v_0 e^{\frac{-bt_{crit}}{M}}$

$e^{\frac{-bt_{crit}}{M}} = Mg - Ma$

$t_{crit} = -\frac{M}{b} \ln(Mg - Ma)$

Go to Question 2 when you're done with this question.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A

$$i) \Sigma F = ma$$

$$F_N = Ma$$

$$\frac{F_N}{M} = a_{\text{cube}}$$

$$ii) \Sigma F = ma$$

$$F_R + F_N = Ma$$

$$-bV + F_N = Ma$$

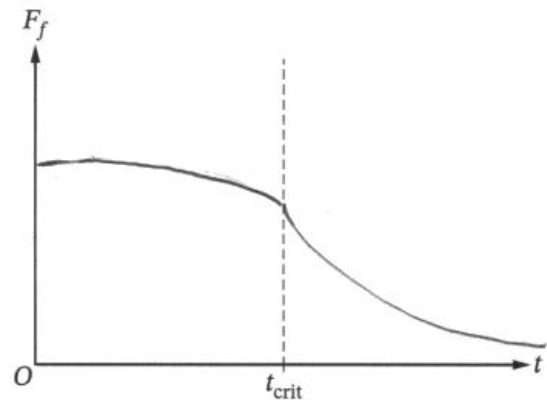
$$-bV + F_N = M \frac{dv}{dt}$$

$$dt = M \frac{dv}{-bV + F_N}$$

$$\int_0^t \frac{dt}{M} = \int_{v_0}^{v_f} \frac{dv}{-bV + F_N}$$

$$\frac{1}{M} t = \ln(-bV_0 + F_N)$$

Figure 2



PART B

t_{crit} is when box starts to slide

$$\Sigma F = ma$$

$$F_f < F_g \text{ when box slides}$$

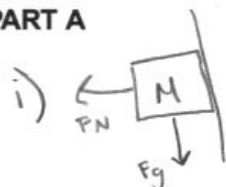
$$\mu_s F_N = F_f$$

$$Mg = \mu_s F_N$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A



$$\Sigma F = F_N = Ma$$

$$a = \frac{F_N}{M}$$

ii) $\Sigma F = ma$

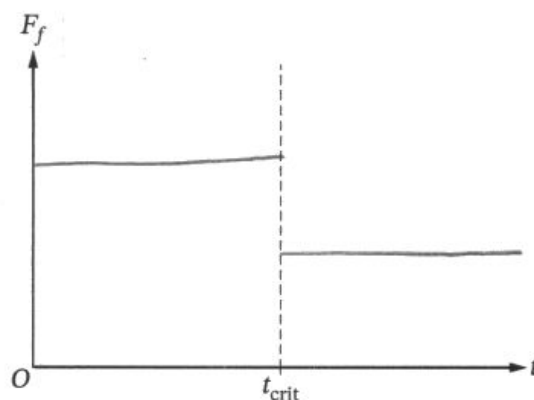
$$F_N = M \frac{dv}{dt}$$

$$\int_0^+ \frac{F_N}{M} dt = \int_{v_0}^v dv$$

$$\frac{F_N}{M} t = v - v_0$$

$$F_N = \frac{M}{t} (v - v_0)$$

Figure 2



PART B



$$\Sigma F = ma$$

$$F_f - F_g = Ma$$

$$Ng_{us} - Mg = M \frac{dv}{dt}$$

$$g_{us} - g = \frac{dv}{dt}$$

$$\int_0^{t_{crit}} g_{us} - g dt = \int_{v_0}^v dv$$

$$g_{us} t - g t \Big|_0^{t_{crit}} = v_0$$

$$t_{crit} (g_{us} - g) = v_0$$

$$t_{crit} = \frac{v_0}{g_{us} - g}$$

Go to Question 2 when you're done with this question.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A

$$i \quad \Sigma \vec{F} = m\vec{a}$$

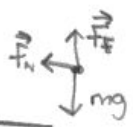
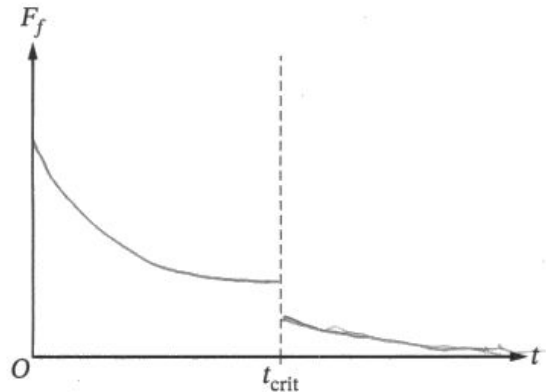
$$F_N = ma \quad \text{so} \quad a = \frac{F_N}{M}$$


Figure 2



$$ii \quad \Sigma \vec{F} = m\vec{a} \quad F_N - bv = ma$$

Force of Friction = mg

$$F_{\text{net}} \Delta t = \Delta P \rightarrow (F_N - bv)\epsilon = m(-v_0)$$

$$\text{So } F_N(t) = -\frac{mv_0}{t} + bv_0$$

PART B

$$\Sigma \vec{F} = m\vec{a} \quad \text{Newton's Laws}$$

$$mg - F_f = ma \quad \text{At } t_{\text{crit}}, mg > F_s$$

$$\text{So } t_{\text{crit}} = \frac{\mu_s Mg - bv_0}{M}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1: Version J

PART A

i) $\sum F = ma$
 $f_R \leftarrow \bullet \rightarrow f_N$
 $\downarrow f_g$
 $\sum F = \frac{f_N}{m} = -\frac{f_N}{m} \rightarrow a = -\frac{f_N}{m}$

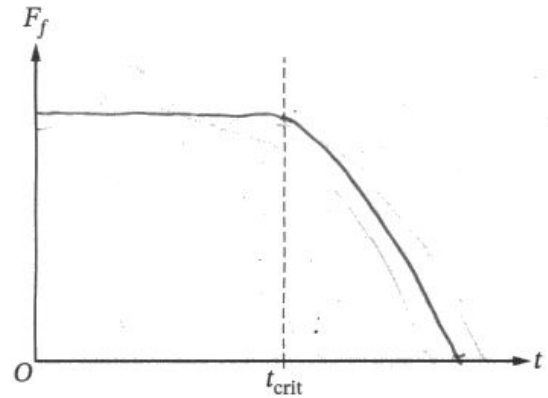
ii) $\sum F = ma$

$f_N - f_R = 0$

$f_N = bV$

$f_N = bV$

Figure 2



PART B

$f_R > f_N$

$f_g > f_f$

$\frac{1}{2} M v_0^2 =$

$t_{crit} \Rightarrow f_f = 0$

Mathematical Routines (MR)

Sample: 1A

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 1

MR Part A Point 4 Score: 1

MR Part A Point 5 Score: 1

MR Part A Point 6 Score: 1

MR Part A Point 7 Score: 1

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 1

MR Part B Point 3 Score: 1

Score: 10

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law.
- Point A3 was earned. The response indicates that the magnitude of the acceleration is $bv/2M$.
- Point A4 was earned. The response separates dv and dt variables.
- Point A5 was earned. The response includes an exponential expression for F_N that evaluates to zero at infinite time.

A. iii

- Point A6 was earned. The response includes a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was earned. The response includes a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law.

- Point B2 was earned. The response indicates that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} .
- Point B3 was earned. The response includes a substitution of the expression for F_N derived in part A into the expression for static friction.

Sample: 1B

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 1

MR Part A Point 4 Score: 1

MR Part A Point 5 Score: 1

MR Part A Point 6 Score: 0

MR Part A Point 7 Score: 1

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 1

MR Part B Point 3 Score: 1

Score: 9

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law.
- Point A3 was earned. The response indicates that the magnitude of the acceleration is $bv/2M$.
- Point A4 was earned. The response separates dv and dt variables.
- Point A5 was earned. The response includes an exponential expression for F_N that evaluates to zero at infinite time.

A. iii

- Point A6 was not earned. The response does not include a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was earned. The response includes a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law.

- Point B2 was earned. The response indicates that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} .
- Point B3 was earned. The response includes a substitution of the expression for F_N derived in part A into the expression for static friction.

Sample: 1C

MR Part A Point 1 Score: 0

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 0

MR Part A Point 4 Score: 1

MR Part A Point 5 Score: 1

MR Part A Point 6 Score: 1

MR Part A Point 7 Score: 1

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 1

MR Part B Point 3 Score: 1

Score: 8

A. i

- Point A1 was not earned. The response does not include a correct expression for the magnitude of the horizontal acceleration of the cube. Specifically, the response incorrectly attempts to solve for the vertical acceleration instead of the horizontal acceleration.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, this response includes a net force equation in the form of $\Sigma F = ma$ and is followed by a multistep derivation.
- Point A3 was not earned. The response does not indicate that the magnitude of the acceleration is $bv/2M$. Specifically, the equation $-bv = m \frac{dv}{dt}$ is given, which is missing the $2M$ variable.
- Point A4 was earned. The response separates dv and dt variables. Specifically, this response shows $a = dv/dt$ and then separates it onto each side with the integral.
- Point A5 was earned. The response includes an exponential expression for F_N that evaluates to zero at infinite time. Specifically, the response includes $F_N \sim e^{\frac{-bt}{m}}$.

A. iii

- Point A6 was earned. The response includes a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.

- Point A7 was earned. The response includes a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, the response includes $\Sigma F = ma$ and then follows with $\Sigma F = 0$.
- Point B2 was earned. The response indicates that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} . Specifically, the response includes $\mu_s F_N = mg$.
- Point B3 was earned. The response includes a substitution of the expression for F_N derived in part A into the expression for static friction. Responses that omit the subscript on μ_s may earn this point. Specifically, the response correctly substitutes the expression for F_N derived in part A into the expression for static friction.

Sample: 1D

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 0

MR Part A Point 4 Score: 1

MR Part A Point 5 Score: 1

MR Part A Point 6 Score: 1

MR Part A Point 7 Score: 1

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 0

MR Part B Point 3 Score: 0

Score: 7

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube. Specifically, the response includes $a = -F_N/M$, and it correctly applies the absolute value to reflect the magnitude, as shown in the prompt.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, this response includes a net force equation in the form of $\Sigma F = ma$ and is followed by a multistep derivation.
- Point A3 was not earned. The response does not indicate that the magnitude of the acceleration is $bv/2M$. Specifically, the equation $m \frac{dv}{dt} = -bv$ in the response is missing the $2M$ variable.
- Point A4 was earned. The response separates dv and dt variables. Specifically, this response shows $a = dv/dt$ and then separates it.
- Point A5 was earned. The response includes an exponential expression for F_N that evaluates to zero at infinite time.

A. iii

- Point A6 was earned. The response includes a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was earned. The response includes a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, the response includes $\Sigma F=ma$ and then follows with a multistep derivation.
- Point B2 was not earned. The response does not indicate that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} .
- Point B3 was not earned. The response does not include a substitution of the expression for F_N derived in part A into the expression for static friction. Specifically, the response substitutes the expression for F_N derived in part A into the expression for kinetic friction instead of static friction.

Sample: 1E

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 0

MR Part A Point 4 Score: 1

MR Part A Point 5 Score: 0

MR Part A Point 6 Score: 0

MR Part A Point 7 Score: 1

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 1

MR Part B Point 3 Score: 0

Score: 6

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, this response includes a net force equation in the form of $\Sigma F = ma$ and is followed by a multistep derivation.
- Point A3 was not earned. The response does not indicate that the magnitude of the acceleration is $bv/2M$. Specifically, the equation is missing the $2M$ variable and includes the normal force.
- Point A4 was earned. The response separates dv and dt variables. Specifically, this response shows $a = dv/dt$ and then separates it onto each side with the integral.
- Point A5 was not earned. The response does not include an exponential expression for F_N that evaluates to zero at infinite time. Specifically, the response does not indicate that $F_N \sim e^{-t}$.

A. iii

- Point A6 was not earned. The response includes a sketch of a non-horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was earned. The response includes a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, the response includes $\Sigma F = ma$ and then follows with a multistep derivation.
- Point B2 was earned. The response indicates that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} . Specifically, the response includes both $\mu_s F_N = F_f$ and $Mg = \mu_s F_N$, indicating the net force is zero.
- Point B3 was not earned. The response does not include a substitution of the expression for F_N derived in part A into the expression for static friction.

Sample: 1F

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 0

MR Part A Point 4 Score: 1

MR Part A Point 5 Score: 0

MR Part A Point 6 Score: 1

MR Part A Point 7 Score: 0

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 0

MR Part B Point 3 Score: 0

Score: 5

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law. Specifically, the response included $\Sigma F = ma$ and includes a multistep derivation.
- Point A3 was not earned. The response does not indicate that the magnitude of the acceleration is $bv/2M$. Specifically, the response never substitutes $Fr = -bv$ into their equation to obtain the $bv/2M$ equation.
- Point A4 was earned. The response separates dv and dt variables.
- Point A5 was not earned. The response does not include an exponential expression for F_N that evaluates to zero at infinite time. Specifically, the response never indicates $F_N \sim e^{-t}$.

A. iii

- Point A6 was earned. The response includes a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was not earned. The response does not include a concave up curve that never increases for $t > t_{\text{crit}}$.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law.
- Point B2 was not earned. The response does not indicate that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} .
- Point B3 was not earned. The response does not include a substitution of the expression for F_N derived in part A into the expression for static friction. Specifically, the response never uses the normal force in this part of the problem.

Sample: 1G

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 0

MR Part A Point 4 Score: 0

MR Part A Point 5 Score: 0

MR Part A Point 6 Score: 0

MR Part A Point 7 Score: 1

MR Part B Point 1 Score: 1

MR Part B Point 2 Score: 0

MR Part B Point 3 Score: 0

Score: 4

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law.
- Point A3 was not earned. The response does not indicate that the magnitude of the acceleration is $bv/2M$. Specifically, the response uses M instead $2M$.
- Point A4 was not earned. The response does not separate dv and dt variables. Specifically, the response doesn't show that $a = dv/dt$, so the variables cannot be separated.
- Point A5 was not earned. The response includes an exponential expression for F_N that evaluates to zero at infinite time. Specifically, the response doesn't include expression $F_N \sim e^{-t}$.

A. iii

- Point A6 was not earned. The response does not include a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was earned. The response includes a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$. Specifically, even though the curve is very slight, it approaches zero asymptotically.

B.

- Point B1 was earned. The response includes a multistep derivation that includes Newton's second law.
- Point B2 was not earned. The response does not indicate that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} . Specifically, the response shows $mg > F_s$.
- Point B3 was not earned. The response does not include a substitution of the expression for F_N derived in part A into the expression for static friction. Specifically, the response doesn't include F_N in the derivation.

Sample: 1H

MR Part A Point 1 Score: 1

MR Part A Point 2 Score: 1

MR Part A Point 3 Score: 0

MR Part A Point 4 Score: 0

MR Part A Point 5 Score: 0

MR Part A Point 6 Score: 1

MR Part A Point 7 Score: 0

MR Part B Point 1 Score: 0

MR Part B Point 2 Score: 0

MR Part B Point 3 Score: 0

Score: 3

A. i

- Point A1 was earned. The response includes a correct expression for the magnitude of the horizontal acceleration of the cube.

A. ii

- Point A2 was earned. The response includes a multistep derivation that includes Newton's second law.
- Point A3 was not earned. The response does not indicate that the magnitude of the acceleration is $bv/2M$. Specifically, the response uses M instead $2M$.
- Point A4 was not earned. The response does not separate dv and dt variables. Specifically, the response doesn't show that $a = dv/dt$, so the variables cannot be separated.
- Point A5 was not earned. The response does not include an exponential expression for F_N that evaluates to zero at infinite time. Specifically, the response doesn't include the expression $F_N \sim e^{-t}$.

A. iii

- Point A6 was earned. The response includes a sketch of a horizontal line from $t=0$ to $t=t_{\text{crit}}$.
- Point A7 was not earned. The response does not include a concave up curve that never increases for $t > t_{\text{crit}}$ and is earned regardless of the discontinuity at $t=t_{\text{crit}}$. Specifically, this incorrect curve is concave down.

B.

- Point B1 was not earned. The response does not include a multistep derivation that includes Newton's second law.
- Point B2 was not earned. The response does not indicate that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit} . Specifically, the response shows $F_g > F_s$.
- Point B3 was not earned. The response does not include a substitution of the expression for F_N derived in part A into the expression for static friction.